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Bowling is a popular sport among people all over the world. In the United States, the most popular form of bowling is ten-pin bowling. Other forms of bowling include five-pin bowling, nine-pin skittles, and duckpin bowling, to name a few.
Chapter 8 Overview

This chapter develops an understanding of the Distributive Property through real-world situations, manipulatives, and analysis of student work. An emphasis is placed on extending and applying properties of operations to generate equivalent expressions, and to determine when two expressions are equivalent.

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## Skills Practice Correlation for Chapter 8

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Learning Goals

In this lesson, you will:

- Predict the next term in a sequence.
- Write numerical and algebraic expressions.

Key Terms

- sequence
- term

Essential Ideas

- A term is a number, a variable, or a product of numbers or variables. It is used to describe the components of a sequence and the components of an algebraic expression.
- A sequence is a pattern involving an ordered arrangement of numbers, geometric figures, letters, or other objects. Each sequence is composed of terms or members.
- An algebraic expression is a mathematical phrase involving at least one variable and sometimes numbers and operation symbols.
- Algebraic expressions can be written to represent sequences, contexts and mathematical word phrases.

Texas Essential Knowledge and Skills for Mathematics

Grade 6

(7) Expressions, equations, and relationships. The student applies mathematical process standards to develop concepts of expressions and equations. The student is expected to:

- distinguish between expressions and equations verbally, numerically, and algebraically
Overview
Students make sense of algebraic expressions by interacting with sequences, contexts, and mathematical terminology used to generate them. Students generate sequences from a context and diagram, describe the pattern in words, and will then represent the sequence as an algebraic expression. Layers of scaffolding are removed, leading to students writing algebraic expressions directly from sequences.

The remainder of the lesson focuses on algebraic expressions being viewed as a series of terms linked together by operation signs. Students decompose given algebraic expressions by stating the number of terms in each algebraic expression and listing the terms. They will practice composing algebraic expressions from sentences in context and verbal phrases written with mathematical terminology.
Warm Up

Rewrite each statement using symbols.

1. Fourteen more than six
   \[ 6 + 14 \]

2. Six more than fourteen.
   \[ 14 + 6 \]

3. Seven less than thirteen.
   \[ 13 - 7 \]

4. Thirteen less than seven.
   \[ 7 - 13 \]

5. Twenty-three subtracted from thirty.
   \[ 30 - 23 \]

6. Thirty subtracted from twenty-three.
   \[ 23 - 30 \]

7. The quotient of twelve divided by four.
   \[ \frac{12}{4} \]

8. The quotient of four divided by twelve.
   \[ \frac{4}{12} \]

9. One-fourth of twenty-eight.
   \[ \frac{1}{4} \times 28 \]

10. Fifteen times nine.
    \[ 15 \times 9 \]

11. Which expressions can be written in another order?
    Numbers 1, 2, 9 and 10.
Have you ever wondered how many parts make up a car? Sure, you can see some parts of a car like the tires, the steering wheel, and the windshield. But what about the other parts you cannot see like the brakes or the fuel filter. Actually, it is estimated that there are roughly 17,000 to 20,000 parts in an average car.

Can you think of some car parts that you cannot see? Do you think it is helpful for car manufacturers to know how many parts are needed for each vehicle they make?
Problem 1
Sequences and terms of a sequence are defined within the context of a problem situation. Students generate sequences from a context and diagram, describe the pattern in words and represent the sequence as an algebraic expression. They then repeat the process using another context, this time without a diagram. Lastly, they will write algebraic expressions to represent sequences directly from the sequences, without contexts or diagrams.

Grouping
Have students complete Questions 1 through 8 with a partner. Then share the responses as a class.

Share Phase, Questions 1 and 2
- Describe what is happening in the problem.
- How many wheels are on one car?
- How many wheels are on two cars?
- How many wheels are on three cars?
- Describe any patterns you see.
- How did you calculate number of wheels on this train?
- How could you calculate the number of wheels without drawing a picture?

Problem 1  Building Trains
Josh likes building trains out of train parts. Each train car needs four wheels. Josh begins by building a train with one car, then two cars, and finally three cars as shown.

1. Describe how many total wheels Josh will need to build a train with 4 cars. How many total wheels will he need to build a train with 5 cars?
   Josh will need 16 wheels to build a train with four cars.
   Josh will need 20 wheels to build a train with 5 cars.

A sequence is a pattern involving an ordered arrangement of numbers, geometric figures, letters, or other objects. The number of train cars and the number of wheels needed form sequences of numbers.

2. Write two sequences using Josh’s train cars.
   Number of cars: 1, 2, 3, 4,...
   Number of wheels: 4, 8, 12, 16,...

- What mathematical operations could you use to calculate the number of wheels?
- What is a sequence?
Share Phase,
Questions 3 through 8

- What is a term in a sequence?
- What other meanings are there for the word “term”?
- List a sequence of 5 terms that are consecutive odd numbers beginning with 1.
- List a sequence of 4 terms that are multiples of seven beginning with 7.
- What quantity stays the same in Josh’s train building situation?
- What other methods could you use to determine the number of wheels for each train?
- What methods could you use to calculate the number of wheels on a train with 100 cars? Is one method faster than the others? If so, why?
- Describe the pattern relating the number of cars to the number of wheels.

Misconception
When students see a pattern increasing by a constant value, such as in Question 8, they often erroneously write the expression using addition instead of multiplication. In this case, a common wrong answer would be \( n + 4 \) rather than \( 4n \). Strategies to help students reason through the correct process include: (1) asking them to substitute values for \( n \) into their expression to see that it will not generate the correct sequence; (2) asking them to calculate the value of a higher number term, such as 100, that would dissuade them from using the previous term’s value and repeated addition; (3) discussing the fact that the relationship that is being described is the relationship between the term number and the term value, not the relationship between the term number and the previous term; and (4) suggesting to convert the sequence to horizontal or vertical table listing the terms numbers as well, which makes the relationship between term number and term explicit.

Each number of cars in the first sequence and each number of wheels in the second sequence form the terms, or members, of the sequence. A term is a number, a variable, or a product of numbers and variables. The first term is the first object or number in the sequence. The second term is the second object or number in the sequence, and so on.

3. What is the first term in the second sequence?
   The first term is 4.

4. What is the fifth term in the first sequence?
   The fifth term is 5.

5. What are the two quantities that change in Josh’s train-building situation?
   The two quantities that change are the number of cars and the number of wheels.

6. Explain how you would determine the number of wheels needed to make a train with 6 cars, 7 cars, and 8 cars.
   I know that five cars require a total of 20 wheels. So, I can just keep adding 4 to the total of 20 wheels. Each time I add another car, I add 4 to that total number of wheels. So for six cars, I would add 4 to 20; for seven cars I would add 6, or 4 + 4, and for eight cars, I would add 12, or 4 + 4 + 4.

7. Describe in your own words how to calculate the number of wheels needed if the train has any number of cars.
   I would multiply the number of cars by 4 to calculate the total number of wheels.

8. Write an algebraic expression to represent the number of wheels needed to build a train with an unknown number of cars.
   \( 4n \)
9. Josh would like to add more trains to his collection. The cost of each car is $8.
   a. Write a sequence that represents the cost of buying 1 car, 2 cars, 3 cars, 4 cars, and 5 cars.
      Cost of cars: $8, $16, $24, $32, $40

   b. Describe the mathematical process that is being repeated to determine the total cost for a set of cars.
      I added 8 to the sum each time to determine the cost of each new set of cars.

   c. What are the variable quantities, or the quantities that are changing, in this situation? Include the units that are used to measure these quantities.
      The variable quantities in this situation are the number of cars to purchase and the cost, in dollars, to buy the cars.

   d. What quantity remains constant? Include the units that are used to measure this quantity.
      The cost of each train car remains constant at $8 per car.

   e. Which variable quantity depends on the other variable quantity?
      The total cost of the cars depends on how many cars will be purchased.

   f. Write an algebraic expression for the total cost, in dollars, for a train set with \( n \) cars. Rewrite the first five terms of the sequence.
      \( 8n \), or 8 times the number of cars
      Cost of cars: $8, $16, $24, $32, $40
Grouping
Have students complete Question 10 with a partner. Then share the responses as a class.

Share Phase, Question 10
- Describe the pattern in words.
- How can you check that your algebraic expression is correct?
- Why should you always begin by substituting the value of one to generate the sequence?
- What operation can you use to represent repeated addition by the same number?

Note
It is expected that students will struggle with Question 10. The purpose of these questions is to demonstrate that all sequences are not generated the same way. Students will have many opportunities throughout the course to become proficient representing a pattern algebraically.

Problem 2
Algebraic expressions are viewed as a series of terms linked together by operation signs. Students decompose given algebraic expressions by stating the number of terms in each algebraic expressions and listing the terms. They will practice composing algebraic expressions from sentences in context and verbal phrases written with mathematical terminology. The final series of problems is more open-ended providing lists of criteria from which to construct algebraic expressions.

10. Write an algebraic expression to represent each sequence.
   a. 2, 4, 6, 8, ...
      \[2n\]
   b. 1, 3, 5, 7, ...
      \[2n - 1\]
   c. 3, 5, 7, 9, ...
      \[2n + 1\]
   d. 2, 4, 8, 16, ...
      \[2^n\]
   e. 101, 102, 103, 104, ...
      \[n + 100\]
1. Let’s consider two algebraic expressions:
   \[ 8 + 5x \quad \text{and} \quad 8 - 5x \]
   a. Identify the number of terms in each algebraic expression.
      Each algebraic expression has 2 terms.
   
   b. Identify the operation in each algebraic expression.
      The first expression contains addition, and the second expression contains subtraction.
   
   c. Identify the terms in each algebraic expression.
      The terms are the same in both expressions. The first term is 8, and the second term is 5x.
   
   d. What is the same in both expressions?
      The number of terms, and the terms themselves are the same.
   
   e. What is different in the expressions?
      The operations are different.

2. Identify the number of terms, and then the terms themselves for each algebraic expression.
   a. \[ 4 - 3x \]
      2 terms
      The first term is a constant 4 and the second term is 3x.
   
   b. \[ 4a - 9 + 3a \]
      3 terms
      The first term is 4a, the second term is the constant 9, and the third term is 3a.
   
   c. \[ 7b - 9x + 3a - 12 \]
      4 terms
      The first term is 7b, the second term is 9x, the third term is 3a, and the fourth term is the constant 12.

Grouping
In preparation for Question 1, have English Language Learners design one or more concept maps using the following words: sequence, term, quantity, algebraic expression, operation, number, variable, and constant. Invite students to Pair/Share their maps. Stimulate conversation by having students finish the following sentence stems:
   • The words I used for this map were __________.
   • The word ________ is mapped to the word ________ because ________.

Discuss Phase, Worked Example
• Compare the definition of “term” when used with a sequence and when used with an algebraic expression.
• How can you tell how many terms are in an algebraic expression?
• Rewrite the expression with the constant 7 written first. How can you tell what operation signs to use in front of each term?

Grouping
Have students complete Questions 1 and 2 with a partner. Then share the responses as a class.

Share Phase, Questions 1 and 2
Rewrite each expression with the constant 5 written first.
Grouping
Have students complete Questions 3 through 5 with a partner. Then share the responses as a class.

Share Phase, Question 3
• How many miles did Diego drive in 1 hour, 2 hours, 3 hours and 10 hours?
• Use your algebraic expression to calculate the number of miles Diego could drive in 7 hours.
• How much money was collected if 1 student bought lunch, 2 students bought lunch and 50 students bought lunch?
• Use your algebraic expression to calculate the amount of money collected if 200 students bought lunch.
• How many cookies would each friend receive if she shared the cookies among 2 friends, 3 friends and 6 friends?
• Use your algebraic expression to calculate the amount of money collected if the cookies were shared among 12 friends.
• How much money would Donald have in his account if he withdrew $10, $20 and $30?
• Use your algebraic expression to calculate the amount of money Donald has left in his account if he withdrew $98 dollars.

3. Write an algebraic expression that represents each situation.
   a. Diego drove at a constant rate of 60 miles per hour. How many miles did he drive in $t$ hours?
      \[ 60t \]
   b. The cost of a school lunch is $1.60. How much money was collected if $n$ students bought lunch?
      \[ 1.60n \]
   c. Jackie has 12 cookies. She wants to share them equally among her friends. How many cookies does each friend receive if she shares with $f$ friends?
      \[ \frac{12}{f} \]
   d. Donald has $145 in his savings account. How much is left in his account if he withdraws $d$ dollars?
      \[ 145 - d \]
   e. The cost to rent a storage unit is $125. The cost will be shared equally among a number of people, $p$, who are storing their belongings. How much will each person pay?
      \[ \frac{125}{p} \]
   f. Chairs cost $35 and sofas cost $75. How much would it cost to purchase $x$ chairs and $y$ sofas?
      \[ 35x + 75y \]
   g. Used paperback books cost $6.25 each with a shipping and handling cost of $8.75. What is the cost of $x$ books?
      \[ 6.25x + 8.75 \]

• For Question 3, part (e), how did you determine what operation to use? What previous question is this question similar to?
• Use your algebraic expression to calculate the amount of money each person would pay if 8 people used the storage unit.
• For Question 3, part (f), explain the meaning of each term in the algebraic expression and why you chose the operation that you did.
• Use your algebraic expression to calculate the cost of one sofa and two chairs.

continued on the next page
• For Question 3, part (g), how did you determine what number should be used as the coefficient of $x$?
• Use your algebraic expression to calculate the cost of 1 book and 10 books.

**Share Phase, Questions 4 and 5**

• How did you decide what operation to use in your algebraic expression?
• Could the algebraic expression be written another way and still be correct? If so, provide another solution.
• For the questions involving subtraction, how did you determine what value should have the minus sign in front of it?
• Explain how you constructed your algebraic expression.
• What mathematical terms did you need to know to construct your expression?
• What are the coefficients in your algebraic expression?

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<td>Write an algebraic expression that represents each word expression.</td>
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<td>a.</td>
<td>the quotient of a number, $n$, divided by 7 $\frac{n}{7}$</td>
</tr>
<tr>
<td>b.</td>
<td>three more than a number, $n$ $n + 3$</td>
</tr>
<tr>
<td>c.</td>
<td>one-fourth of a number, $n$ $\frac{n}{4}$</td>
</tr>
<tr>
<td>d.</td>
<td>fourteen less than three times a number, $n$ $3n - 14$</td>
</tr>
<tr>
<td>e.</td>
<td>six times a number, $n$, subtracted from 21 $21 - 6n$</td>
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<tr>
<td>f.</td>
<td>three more than a number, $n$, added to 21 $n + 3 + 21$</td>
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<tr>
<td>g.</td>
<td>one-fourth of a number, $n$, minus 6 $\frac{1}{4}n - 6$</td>
</tr>
<tr>
<td>h.</td>
<td>Ten times the square of a number, $w$, divided by 12. $\frac{10w^2}{12}$</td>
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5. Construct an algebraic expression for each description.

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<td>a.</td>
<td>There are 2 terms. The first term is a constant added to the second term, which is a product of a number and a variable. Sample answer: $11 + 3x$</td>
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<tr>
<td>b.</td>
<td>There are 4 terms. The first term is a quotient of a variable divided by 11. This is added to a second term, which is a constant. The third term is a second variable multiplied by three-fourths. The third term is subtracted from the first 2 terms. The last term is a different constant added to the other 3 terms. Sample answer: $\frac{x}{11} + 3 - \frac{3}{4}y + 12$</td>
</tr>
<tr>
<td>c.</td>
<td>The cube of a variable subtracted from a constant and then added to the square of the same variable. Sample answer: $12 - x^3 + x^2$</td>
</tr>
<tr>
<td>d.</td>
<td>A number multiplied by the square of a variable minus a number multiplied by the same variable minus a constant. Sample answer: $12x^2 - 7x - 9$</td>
</tr>
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</table>

Be prepared to share your solutions and methods.
Assignment
Use the Assignment for Lesson 8.1 in the Student Assignments book. See the Teacher’s Resources and Assessments book for answers.

Skills Practice
Refer to the Skills Practice worksheet for Lesson 8.1 in the Student Assignments book for additional resources. See the Teacher’s Resources and Assessments book for answers.

Assessment
See the Assessments provided in the Teacher’s Resources and Assessments book for Chapter 8.

Check for Students’ Understanding
1. Ross is saving $3 from his allowance each week.
   a. Write a sequence to represent the amount of money Ross has after each week for the first 5 weeks.
      3, 6, 9, 12, 15
   b. Write an algebraic expression to represent Ross’ savings, in dollars, for $w$ number of weeks.
      $3w$
2. Ross’ twin sister Rosalyn has $10 saved and plans to save $2 per week.
   a. Write a sequence to represent the amount of money Rosalyn has after each week for the first 5 weeks.
      12, 14, 16, 18, 20
   b. Write an algebraic expression to represent Rosalyn’s savings, in dollars, for $w$ number of weeks.
      $10 + 2w$
3. Given the algebraic expression: $8x + 15$
   
   a. Write a sequence of the first five terms generated by this algebraic expression.
      
      $23, 31, 39, 47, 55$
   
   b. What pattern do you see in the sequence?
      
      The values increase by 8 every time.
   
   c. How does this pattern relate to the algebraic expression?
      
      The increase of 8 in the sequence is represented by the multiplication by 8 in the algebraic expression.
   
   d. What is the value of the expression when $x = 11$?
      
      When $x = 11$, the value of the expression is 103.
Essential Ideas

- The Commutative Properties of Addition and Multiplication state that the order in which you add or multiply two or more numbers does not affect the sum or the product.
- The Associative Properties of Addition and Multiplication state that changing the grouping of the terms in an addition problem or a multiplication problem does not change the sum or the product.
- To simplify an expression is to use the rules of arithmetic and algebra to rewrite that expression as simply as possible. An expression is in simplest form if all like terms have been combined.
- Like terms are two or more terms that have the same variable raised to the same power.
- Algebra tiles are used as a model to make sense of simplifying algebraic expressions.

Texas Essential Knowledge and Skills for Mathematics

Grade 6

(7) Expressions, equations, and relationships. The student applies mathematical process standards to develop concepts of expressions and equations. The student is expected to:

(D) generate equivalent expressions using the properties of operations: inverse, identity, commutative, associative, and distributive properties

Materials

Algebra tiles
Overview

Students write an algebraic expression for a given situation. Equivalent forms of this expression are used to introduce the Commutative Properties of Addition and Multiplication. Students then evaluate several numerical expressions using the Order of Operations Rules to conclude that the expressions are equivalent despite being grouped differently; this process is used to introduce the Associative Properties of Addition and Multiplication. Students will simplify numeric expressions using the Order of Operation Rules and the new properties introduced in this lesson.

The terms simplify and like terms are defined as students transfer their thinking back to algebraic expressions. Students will simplify algebraic expressions first by using algebra tiles to make sense of combining like terms, and then by using the rules and properties learned without the scaffolding of the algebra tile models.
Warm Up

1. Simplify: $3 + 4 + 5$
   $12$

2. Simplify: $4 + 5 + 3$
   $12$

3. Simplify: $5 + 3 + 4$
   $12$

4. Compare the answers to Questions 1, 2 and 3.
   The answers are the same.

5. Does the order in which you add numbers change the sum?
   The order in which you add numbers does not change the sum.

6. Simplify: $3 \times 4 \times 5$
   $60$

7. Simplify: $4 \times 5 \times 3$
   $60$

8. Simplify: $5 \times 3 \times 4$
   $60$

9. Compare the answers to Questions 6, 7, and 8.
   The answers are the same.

10. Does the order in which you multiply numbers change the product?
    The order in which you multiply numbers does not change the product.
Learning Goals
In this lesson, you will:
- Use the Associative and Commutative Properties of Addition and Multiplication to simplify expressions.
- Use the Order of Operations.
- Use algebra tiles to simplify algebraic expressions.

Key Terms
- Commutative Property of Addition
- Commutative Property of Multiplication
- Associative Property of Addition
- Associate Property of Multiplication
- Simplify
- Like terms

If you type your papers, you probably use a word processing program. If you blog, you probably use some type of text program. If you surf the net, you use a browser. What do these and many other computer programs have in common? They all use programming.

Computer programming is the ability of writing directions so that the computer can perform the task. Just think about it: every drop-down menu you select, every mouse click on an icon, even when you close out a program requires directions for the computer to perform the task you want.

As with any directions, the order of the directions is important. Just imagine the confusion that might happen if scrolling on a drop-down menu closed out a program. This is why computer programmers must be very careful how they instruct computers to do tasks.

Can you think of other directions that people or machines do to perform tasks? Is the order of the directions important?
Problem 1
Students compute the total cost, write an algebraic expression, identify the variable quantities and identify the constant quantities representing a situation. Four different ways to write the algebraic expression are provided. These expressions are equivalent and are shown as such using the Commutative Properties of Addition and Multiplication. Students use the Order of Operation Rules to evaluate several numerical expressions.

Grouping
Have students complete Questions 1 through 6 with a partner. Then share the responses as a class.

Share Phase, Questions 1 through 6
• Have you ever gone bowling?
• Do you have to rent shoes?
• Why can’t you use your own shoes?
• Do you have to rent shoes twice to bowl two games?
• Is the shoe rental fee charged for each game you bowl?
• Can you buy your own bowling shoes and not rent them?
• Is there a rental fee for using the bowling ball?
• Is the second game less expensive than the first game?

Problem 1  Going Bowling

1. You and your friends are going bowling. It costs $2.50 to rent shoes, and it costs $3.50 to bowl one game.
   a. How much will it cost you to rent shoes and bowl 1 game?
      \[2.5 + 3.5(1) = 6\]
      It will cost me $6 to rent shoes and bowl 1 game.
   b. How much will it cost you to rent shoes and bowl 2 games?
      \[2.5 + 3.5(2) = 9.50\]
      It will cost me $9.50 to rent shoes and bowl 2 games.
   c. How much will it cost you to rent shoes and bowl 3 games?
      \[2.5 + 3.5(3) = 13\]
      It will cost me $13.00 to rent shoes and bowl 3 games.

2. Describe how you calculated the total cost for each situation.
   I multiplied the number of games bowled by $3.50, and then I added $2.50 to the product for the cost to rent shoes.

3. What are the variable quantities in this problem?
   The variable quantities are the number of games bowled, and the total cost to bowl the games.

4. What are the constant quantities in this problem?
   The constant quantities are the cost to rent shoes, and the cost of each game.

5. What variable quantity depends on the other variable quantity?
   The total cost of bowling depends on the number of games bowled.

6. Write an algebraic expression to determine the total cost to rent shoes and bowl games. Let \(g\) represent the unknown number of games bowled.
   \[2.5 + 3.5g\]

• Explain the meaning of each term of your algebraic expression as it relates to the context.
• What is another way the algebraic expression could be written?
• Use your algebraic expression to determine the cost to rent shoes and bowl 6 games.
Grouping
• Ask a student to read the information prior to Question 7 aloud. Discuss the worked example and definitions as a class.
• Have students complete Question 7 independently. Then share the responses as a class.

Discuss Phase, Worked Example and Definitions
• How do the Commutative Properties of Addition and Multiplication relate to the meaning of the word “commute”?
• Explain what the algebraic statement \( a + b = b + a \) means.
• Substitute numbers for the variables \( a \) and \( b \) to demonstrate the meaning of the algebraic statement.
• Is the algebraic statement \( a - b = b - a \) true or false? Provide an example to demonstrate your response.
• Explain what the algebraic statement \( a \times b = b \times a \) means.
• Substitute numbers for the variables \( a \) and \( b \) to demonstrate the meaning of the algebraic statement.
• Is the algebraic statement \( a \div b = b \div a \) true or false? Provide an example to demonstrate your response.

There are several ways to write an algebraic expression to represent the total cost to rent shoes and bowl games.

Let \( g \) represent the number of games bowled.

Expression 1  Expression 2  Expression 3  Expression 4
2.50 + 3.50g  2.50 + g(3.50)  3.50g + 2.50  g(3.50) + 2.50

These expressions are equivalent because they express the same relationship.

The Commutative Properties of Addition and Multiplication state that the order in which you add or multiply two or more numbers does not affect the sum or product.

The Commutative Property of Addition states that changing the order of two or more terms in an addition problem does not change the sum.
For any numbers \( a \) and \( b \), \( a + b = b + a \).

The Commutative Property of Multiplication states that changing the order of two or more factors in a multiplication problem does not change the product.
For any numbers \( a \) and \( b \), \( a \times b = b \times a \).

Expressions 1 and 3 demonstrate the Commutative Property of Addition. These expressions are equivalent, even though the order of the terms is reversed.

Expressions 1 and 2 demonstrate the Commutative Property of Multiplication. These expressions are equivalent, even though the factors in the second terms are reversed.

7. Given the four expressions shown:
   a. state one other example of the Commutative Property of Addition.
       Expressions 2 and 4 give an example of the Commutative Property of Addition.

   b. state one other example of the Commutative Property of Multiplication.
       Expressions 3 and 4 give an example of the Commutative Property of Multiplication.
Grouping
- Ask a student to read the information prior to Question 8 aloud. Discuss the information as a class.
- Have students complete Question 8 with a partner. Then share the responses as a class.

Share Phase, Question 8
- Did you multiply or subtract first?
- Would you get the same answer if you performed the operations in a different order?
- Did you multiply or add first?
- What do the rules for the order of operations say you should do first?
- How did you get that answer?

When determining the total cost to rent shoes and bowl games, you used the Order of Operations. Recall that these rules ensure that the order in which numbers and operations are combined is the same for everyone. You should remember that when evaluating any expression, you must use the Order of Operations.

8. Evaluate each numerical expression using the Order of Operations. Show all your work.
   a. \(22 - 4(3)\)
      \[
      \begin{array}{c}
      22 - 12 \\
      10
      \end{array}
      \]
   b. \(9(3) + 4(5)\)
      \[
      \begin{array}{c}
      27 + 20 \\
      47
      \end{array}
      \]
   c. \(60 - 3^2\)
      \[
      \begin{array}{c}
      60 - 9 \\
      51
      \end{array}
      \]
   d. \((8 - 5)^2 + 5^2\)
      \[
      \begin{array}{c}
      (3)^2 + 5^2 \\
      (3)^2 + 25 \\
      6 + 25 \\
      31
      \end{array}
      \]
   e. \(7(12) - 2(3 + 2)\)
      \[
      \begin{array}{c}
      7(12) - 2(5) \\
      84 - 10 \\
      74
      \end{array}
      \]
   f. \((10 - 6) + 2^2\)
      \[
      \begin{array}{c}
      4 + 2^2 \\
      4 + 4 \\
      8
      \end{array}
      \]
Problem 2
Students evaluate several numerical expressions to conclude that the expressions are equivalent. The Associative Properties of Addition and Multiplication are then introduced. Students will analyze student work and use these properties. They then will write equivalent numerical expressions and compute the sum or product of each expression.

Grouping

- Have students complete Questions 1 through 3 with a partner. Then share the responses as a class.
- Ask a student to read the definitions following Question 3 aloud. Discuss the content as a class.

Share Phase, Questions 1 through 3

- What do you notice about the parenthesis with respect to the equivalent expressions?
- Is it possible to regroup the expression in a different way?
- Is the order in which it is multiplied important?
- Is the order in which it is added important?
- In each case, was one order of completion simpler than the other? If so, explain.

Discuss Phase, Definitions

- How do the Associative Properties of Addition and Multiplication relate to the meaning of the word “associate”?
- What do you notice about the parenthesis with respect to the equivalent expressions?
- Explain what the algebraic statement \((a + b) + c = a + (b + c)\) means. Substitute numbers for the variables to demonstrate the meaning.

Problem 2  More Properties

1. Evaluate each numerical expression using the Order of Operations.

<table>
<thead>
<tr>
<th>4 + (17 + 3)</th>
<th>4 + (17 + 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>21 + 3</td>
<td>4 + 20</td>
</tr>
<tr>
<td>24</td>
<td>24</td>
</tr>
<tr>
<td>(19 + 42) + 8</td>
<td>19 + (42 + 8)</td>
</tr>
<tr>
<td>61 + 8</td>
<td>19 + 50</td>
</tr>
<tr>
<td>69</td>
<td>69</td>
</tr>
<tr>
<td>(16 \times 5) \times 2</td>
<td>16(5 \times 2)</td>
</tr>
<tr>
<td>80(2)</td>
<td>16(10)</td>
</tr>
<tr>
<td>160</td>
<td>160</td>
</tr>
<tr>
<td>(12 \times 25) \times 4</td>
<td>12(25 \times 4)</td>
</tr>
<tr>
<td>(300)4</td>
<td>12(100)</td>
</tr>
<tr>
<td>1200</td>
<td>1200</td>
</tr>
</tbody>
</table>

2. Compare the two expressions in each row. What do you notice?
   - The expressions are equal. I got the same result.

3. How do the expressions in the first column compare to those in the second column?
   - The terms or factors have been regrouped.

   The **Associative Property of Addition** states that changing the grouping of the terms in an addition problem does not change the sum. For any numbers \(a, b,\) and \(c,\)
   \[(a + b) + c = a + (b + c).
   
   The **Associative Property of Multiplication** states that changing the grouping of the factors in a multiplication problem does not change the product. For any numbers \(a, b,\) and \(c,\)
   
   \[(a \times b) \times c = a \times (b \times c).
   
   The Commutative and Associative Properties of Addition and Multiplication are very powerful properties. For any expression involving only addition, they allow you to change the order of addends and to add parentheses whenever you want. For any expression involving only multiplication, you can also change the order of the factors and include parentheses whenever you want.
Discuss Phase, Definitions

• Is the algebraic statement 
\((a - b) - c = a - (b - c)\) true or false? Provide an example to demonstrate your response.

• Explain what the algebraic statement 
\((a \times b) \times c = a \times (b \times c)\) means. Substitute numbers for the variables to demonstrate the meaning.

• Is the algebraic statement 
\((a + b) + c = a + (b + c)\) true or false? Provide an example to demonstrate your response.

• Explain the difference between the Commutative Property and the Associative Property.

Grouping

Have students complete Questions 4 and 5 with a partner. Then share the responses as a class.

Share Phase, Questions 4 and 5

• Explain the steps in Corrine’s mind for her to write the first line of her work.

• What property allowed her to write the numbers in a different order?

• What property allowed her to use parentheses and group the numbers instead of calculating the sum in the order it was presented?

• Why do you think Corrine solved the problem this way?

• Do you think Benjamin could have solved this problem mentally using his method? Explain.

• Is there another way to group the terms? Would this change the answer?

• How many different ways can these terms in the expression be grouped?

• Why do we need to use parentheses? Do you always need to use parentheses?

• How many sets of parenthesis can be used in one expression?

Benjamin and Corinne are both trying to determine this sum. 
\[22 + 17 + 3 + 8\]

Their methods are shown.

4. Describe the differences between the methods Benjamin and Corinne used to calculate the sum.

Benjamin calculated the sum by adding each number in order. Corinne used the Associative Property of Addition and grouped terms so that calculations might be simpler.

5. Determine each sum by writing an equivalent numerical expression using the Commutative and Associative Properties.

\[
\begin{align*}
\text{a.} & \quad 5 + 7 + 5 \\
& \quad (5 + 5) + 7 \\
& \quad 12 + 7 \\
& \quad 19
\end{align*}
\]

\[
\begin{align*}
\text{b.} & \quad 5 + 13 + 2 \\
& \quad 5 + (13 + 2) \\
& \quad 15 + 2 \\
& \quad 17
\end{align*}
\]

\[
\begin{align*}
\text{c.} & \quad 2 + 21 + 8 + 9 \\
& \quad (2 + 8) + (21 + 9) \\
& \quad 10 + 30 \\
& \quad 40
\end{align*}
\]

\[
\begin{align*}
\text{d.} & \quad 4 + 3 + 6 + 5 \\
& \quad (4 + 3) + (6 + 5) \\
& \quad 7 + 11 \\
& \quad 18
\end{align*}
\]

Use the properties to look for number combinations that make it simpler to add like Corinne did.

Do you think Corrine could have solved this problem mentally using her method? Explain.

Is there another way to group the terms? Would this change the answer?

How many different ways can these terms in the expression be grouped?

Why do we need to use parentheses? Do you always need to use parentheses?

How many sets of parenthesis can be used in one expression?
Problem 3
Definitions for simplify and like terms are given. Students use algebra tiles to explore simplifying algebraic expressions. They will represent several algebraic expressions using algebra tiles and sketch the resulting models. Students will then simplify algebraic expressions using the models they created. Finally, students will simplify several algebraic expressions using properties, order of operation rules, and combining like terms.

Materials
Algebra tiles

Grouping
Ask a student to read the definitions aloud. Discuss the definitions and worked example as a class.

Discuss Phase, Problem 3
- Which expressions contain like terms: $4x + 3x, 3x + 3y, 5x^2 + 8x, 6 + 9, 10m - 3m$?
- Why does it make sense to call the small square tile a “unit tile”?
- Explain why the unit tiles match up with the sides of the $x$ tiles.

Problem 3  Simplifying Algebraic Expressions

The Commutative and Associative Properties can help you simplify algebraic expressions. To simplify an expression is to use the rules of arithmetic and algebra to rewrite that expression with fewer terms. An expression is in simplest form if it contains the fewest terms possible, and if all like terms have been combined.

In an algebraic expression, like terms are two or more terms that have the same variable raised to the same power. The numerical coefficients of like terms can be different.

Let's use algebra tiles to explore simplifying algebraic expressions.

You can represent the algebraic expression $3x + 2$ using algebra tiles.
Grouping

Have students complete Question 1 with a partner. Then share the responses as a class.

Share Phase, Question 1

• How do you know which tile represents ‘x’? What is its length and width?
• How do you know which tile represents ‘y’? What is its length and width?
• How do you know which tile is the unit tile? What is its length and width?
• How are like terms shown using algebra tiles?

1. Represent each algebraic expression using algebra tiles. Then, sketch the model below each algebraic expression.

   a. \(3x + 2\)
   
   b. \(2x + 1\)
   
   c. \(4y + 3\)
   
   d. \(5x\)
   
   e. \(4\)
   
   f. \(2x + 3y\)
Grouping
Have students complete Questions 2 and 3 with a partner. Then share the responses as a class.

Share Phase, Questions 2 and 3
• What operation are you using when you combine like terms?
• Do tiles make it easier to do this problem?
• How can you show the Commutative Property of Addition using algebra tiles?
• How can you show the Associative Property of Addition using algebra tiles?
• Can two people get different answers using the same algebra tiles?
• For which questions would you have difficulty using the algebra tiles to model the problem?
• Why are there more terms in your simplified answer to some problems than others?
• What properties did you use to solve this problem?
• Was your process any different for the problems with fractions, decimals and minus signs?

2. Use your models from Question 1 to write each algebraic expression in simplest form by combining like terms.
   a. \((3x + 2) + (2x + 1)\)  
      \(5x + 3\)
   b. \((3x + 2) + (4y + 3)\)  
      \(3x + 4y + 5\)
   c. \((2x + 1) + (4y + 3)\)  
      \(2x + 4y + 4\)
   d. \((4y + 3) + 4\)  
      \(4y + 7\)
   e. \((3x + 2) + 5x\)  
      \(8x + 2\)
   f. \((4y + 3) + 5x\)  
      \(5x + 4y + 3\)

3. Simplify each algebraic expression.
   a. \(2x + 3x + 4\)  
      \(9x\)
   b. \(\frac{3}{4}x + 2\)  
      \(\frac{3}{4}x + 2\)
   c. \((16x + 4.1) + (10.4x - 2)\)  
      \(26.6x + 2.1\)
   d. \((8x + 12y) + (11x - 8y)\)  
      \(19x + 4y\)
   e. \(55x + 65y + 75\)  
      \(55x + 65y + 75\)
   f. \((2x + 5) + (3y + 2)\)  
      \(2x + 3y + 7\)
   g. \((3y + 2) + (4y + 2)\)  
      \(7y + 4\)
   h. \((x + 1) + (3y + 2) + (4y + 2)\)  
      \(x + 7y + 5\)
Talk the Talk

Students summarize how they know when an algebraic or numerical expression is completely simplified.

Grouping

Have students complete the two questions independently. Then share the responses as a class.

1. How do you know when an algebraic expression is in simplest form?
   An algebraic expression is in simplest form when all like terms have been combined and all constants have been combined. It is written with the fewest possible terms.

2. How do you know when a numerical expression is in simplest form?
   A numerical expression is in simplest form when all the operations have been performed to get a single number or value.

Be prepared to share your solutions and methods.
Follow Up

Assignment
Use the Assignment for Lesson 8.2 in the Student Assignments book. See the Teacher’s Resources and Assessments book for answers.

Skills Practice
Refer to the Skills Practice worksheet for Lesson 8.2 in the Student Assignments book for additional resources. See the Teacher’s Resources and Assessments book for answers.

Assessment
See the Assessments provided in the Teacher’s Resources and Assessments book for Chapter 8.

Check for Students’ Understanding
Simplify each expression.

1. $3x + 4x + 5$
   $7x + 5$
2. $14y + 8 - 3y$
   $11y + 8$
3. $x - 3 + 4x$
   $5x - 3$
4. $13.5y - 10 + 1.5x$
   $13.5y - 10 + 1.5x$
5. $6 + (9 - 2)$
   $13$
6. $(6 + 9) - 2$
   $13$
7. $15 - (8 + 2)$
   $5$
8. $(15 - 8) + 2$
   $9$
9. $(x + 3) + (4x + 7)$
   $5x + 10$
10. $(12.6x + 6.8) + (4x - 3)$
    $16.6x + 3.8$
Learning Goals

In this lesson, you will:

- Simplify algebraic expressions using the Distributive Property.
- Write algebraic expressions using the Distributive Property.
- Model the Distributive Property using algebra tiles.

Key Terms

- Distributive Property of Multiplication over Addition
- Distributive Property of Multiplication over Subtraction
- Distributive Property of Division over Addition
- Distributive Property of Division over Subtraction

Essential Ideas

- The Distributive Property of Multiplication over Addition states if \( a, b \) and \( c \) are any real numbers, then \( a \cdot (b + c) = a \cdot b + a \cdot c \)
- The Distributive Property of Multiplication over Subtraction states if \( a, b \) and \( c \) are any real numbers, then \( a \cdot (b - c) = a \cdot b - a \cdot c \)
- The Distributive Property of Division over Addition states if \( a, b \) and \( c \) are any real numbers, and \( c \neq 0 \), then \( \frac{a + b}{c} = \frac{a}{c} + \frac{b}{c} \).
- The Distributive Property of Division over Subtraction states if \( a, b \) and \( c \) are any real numbers, and \( c \neq 0 \), then \( \frac{a - b}{c} = \frac{a}{c} - \frac{b}{c} \).
- Algebra tiles can be used to illustrate the Distributive Properties.

Texas Essential Knowledge and Skills for Mathematics

Grade 6

(7) Expressions, equations, and relationships. The student applies mathematical process standards to develop concepts of expressions and equations. The student is expected to:

- (C) determine if two expressions are equivalent using concrete models, pictorial models, and algebraic representations
- (D) generate equivalent expressions using the properties of operations: inverse, identity, commutative, associative, and distributive properties

Materials

Algebra tiles
Overview
A context is given that allows students to explore two different methods and expressions for calculating the total area of two rooms. This context and the equivalent expressions generated by it serve to introduce the Distributive Property of Multiplication over Addition. Formal definitions of both the Distributive Property of Multiplication over Addition and the Distributive Property of Multiplication over Subtraction are provided. Algebra tiles are used as a method to make sense of the Distributive Property of Multiplication. Students will simplify expressions using the Distributive Property, the Order of Operation Rules, and combining like terms.

Algebra tiles and student work are then used to make sense of the Distributive Property of Division. The lesson concludes with students simplifying expressions with varying levels of difficulty and stating what form of the Distributive Property was used.
Warm Up

Simplify each expression.
1. \(\frac{14 + 8}{2}\)
   \(\frac{22}{2}\)
   11

2. \(\frac{14 + 8}{2}\)
   \(\frac{22}{2}\)
   11

3. \(\frac{54 - 36}{9}\)
   \(\frac{18}{9}\)
   2

4. \(\frac{54 - 36}{9}\)
   \(\frac{18}{9}\)
   2

5. \(-5(8 - 2)\)
   \(-30\)

6. \((-5 \times 8) - (-5 \times 2)\)
   \(-30\)

7. \((-7 + 2) \times (-5)\)
   \(25\)

8. \((-7) \times (-5) + 2 \times (-5)\)
   \(25\)

9. What do you notice about the answers to Questions 1 and 2?  
The answers are the same.

10. What do you notice about the answers to Questions 3 and 4?  
The answers are the same.

11. What do you notice about the answers to Questions 5 and 6?  
The answers are the same.

12. What do you notice about the answers to Questions 7 and 8?  
The answers are the same.
Do you ever wonder how your school was built? Or have you wondered how your house or apartment building was constructed? Before the actual construction takes place, architects plan what the building will look like, what materials will be needed, and how the new structure will interact with other buildings in the area. In certain cities, there are special requirements so that a building can withstand damage from an earthquake or a hurricane and still remain standing.

Not very long ago, the blueprint was the backbone of the building process. Architects and draftspersons would create the floor plan for buildings, malls, homes, apartment buildings—well, almost any building you can think of. However, as more work is being done on computers, people are no longer using paper blueprints—do you think people will no longer plan buildings as well?
Problem 1
A context is provided that allows students to explore two different methods and expressions for calculating the total area of two rooms. This context and the equivalent expressions generated by it serve to introduce the Distributive Property of Multiplication over Addition. Formal definitions of both the Distributive Property of Multiplication over Addition and the Distributive Property of Multiplication over Subtraction are provided.

Grouping
Have students complete Questions 1 through 4 with a partner. Then share the responses as a class.

Share Phase, Questions 1 through 4
• What shape are the rooms?
• Which side of the room is the length?
• What unit of measurement is used to describe the length?
• What unit of measurement is used to describe the width?
• Which side of the room is the width?
• What do you notice about the width of both rooms?
• How do you compute the area of a rectangle?
• What unit of measurement is used to describe the area?

Problem 1 Installing Carpet

The Lewis family just bought a new house. They will install new carpet in two adjacent rooms before they move in.

1. How would you calculate the total area of the rooms?
Since the rooms are rectangular, I can calculate the area of each room by multiplying the length by the width. Then, I can add the areas together to get the sum of the total area of the two rooms.

Brian
I calculated the area of each room. Then, I added the two areas together to get the area of both rooms:

$(11 \times 13) + (11 \times 7)$

2. Calculate the area using Brian’s expression.

$143 + 77 = 220$
The total area is 220 square feet.

• Is there another way to compute the area of a rectangle?
• How do you compute the total area of two rectangles?
• Was your method of calculating the area the same as Brian’s method or Sara’s method?
• Which method do you prefer?
**Grouping**

Ask a student to read the definitions following Question 4 aloud. Discuss the information as a class.

**Discuss Phase, Distributive Property**

- How do the Distributive Properties relate to the meaning of the word “distribute”?
- Explain what is meant by “multiplication over addition”.
- Explain what is meant by “multiplication over subtraction”.
- Explain what the algebraic statement $a \cdot (b + c) = a \cdot b + a \cdot c$ means.
- Substitute numbers for the variables $a$, $b$, and $c$ to demonstrate the meaning of the algebraic statement.
- Explain what the algebraic statement $a \cdot (b - c) = a \cdot b - a \cdot c$ means.
- Substitute numbers for the variables $a$, $b$, and $c$ to demonstrate the meaning of the algebraic statement.

3. Calculate the area using Sara’s expression.

$$11(20) = 220$$

The total area is 220 square feet.

4. What do you notice about your results using each expression? What does that tell you about the two expressions Brian and Sara wrote?

I calculated the total area to be 220 square feet using both methods. The two expressions are equal.

The two expressions, $(11 \times 13) + (11 \times 7)$ and $11 (13 + 7)$, are equal because of the Distributive Property of Multiplication over Addition.

The **Distributive Property of Multiplication over Addition** states that for any real numbers $a$, $b$, and $c$, $a \cdot (b + c) = a \cdot b + a \cdot c$.

There is also the **Distributive Property of Multiplication over Subtraction**, which states that if $a$, $b$, and $c$ are any real numbers, then $a \cdot (b - c) = a \cdot b - a \cdot c$.

In this chapter, many complex multi-syllable terms are introduced. Break terms into understandable parts so that beginning learners can see their structure. For instance, the Distributive Property of Multiplication over Addition:

- **Distributive property**: a rule for distributing; to distribute means to give a share of something, or to deal out.
- **Of multiplication over addition**: Multiply a number by a sum.

Put each part together with an example. Speak slowly as you write, **The Distributive Property of Multiplication over Addition tells us that two times a sum is the sum of 2 times each number**: $2(x + 3) = 2 \cdot x + 2 \cdot 3$. 

**ELL Tip**

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### Problem 2 Algebra Tiles

Let’s use algebra tiles to explore rewriting algebraic expressions with the Distributive Property.

Consider the expression $5(x + 1)$. This expression has two factors: 5 and the quantity $(x + 1)$. You can use the Distributive Property to rewrite this expression. In this case, multiply the 5 by each term of the quantity $(x + 1)$. The model using algebra tiles is shown.

$$5(x + 1) = 5x + 5$$

This model appears to be just like adding the quantity $x + 1$ five times.

- How do you know which term should be placed horizontally across the top row of the algebra tile model?
- How do you know which term should be placed vertically down the left most column of the algebra tile model?
- What does the dot in the upper most left corner of the algebra tile model represent?
- Which part of the algebra tile model is the product?
- How is the product determined?

### Materials
Algebra tiles

### Grouping
Ask a student to read the information prior to Question 1 aloud. Discuss the worked example as a class.

### Discuss Phase, Worked Example
- Where are the factors represented in the model?

---

**Note**

- Most math problems do not show the multiplication symbol between the first term and the parentheses; instead, the multiplication is implied.
- In some cases, using the Distributive Property first allows for easier calculations and mental math opportunities. For example, $8(20 + 3)$ may be easier to compute mentally as $160 + 24$, rather than $8 \times 23$.

**Problem 2**

Students use algebra tiles to multiply expressions. The example provided simplifies the expression $5(x + 1)$. Students will create similar models as a method to make sense of the Distributive Property. They will then simplify expressions using the Distributive Property, Order of Operation Rules and combining like terms.

**Materials**
Algebra tiles
Grouping
Have students complete Question 1 with a partner. Then share the responses as a class.

Share Phase, Question 1, parts (a) through (c)
- How do you use algebra tiles to multiply expressions?
- Show that the algebra tile model and the process of using the Distributive Property yield the same results.
- How does the algebra tile model relate to the process of using the Distributive Property?

1. Create and sketch a model of each expression using your algebra tiles. Then, rewrite the expression using the Distributive Property.
   a. $3(x + 2)$
   - $3x + 6$
   - $3 \times 1 + 3 \times 1$
   - $1 \times 3 + 1 \times 1$

   b. $4(2x + 1)$
   - $8x + 4$
   - $4 \times 2x + 4 \times 1$
   - $2 \times 4 \times 1 + 2 \times 1$

   c. $2(x + 3)$
   - $2x + 6$
   - $2 \times x + 2 \times 3$
   - $1 \times 2 + 1 \times 3$
Share Phase, Question 1, part (d)

- How is Question 1, part (d) different from the other questions?
- What property can be used to rewrite Question 1, part (d) so that it looks like the other questions?
- Was your process of solving Question 1, part (d) any different from solving the previous questions? If so, explain.
Grouping
Have students complete Question 2 with a partner. Then share the responses as a class.

Share Phase, Question 2
• Explain your process of solving this problem.
• What is different about this problem? How did this difference affect how you solved the problem?
• How can you tell if the resulting algebraic expression is simplified completely?

Misconceptions
• A common mistake occurs when students encounter problems such as 5 + 2(x + 3), where like terms are written in front of the parentheses. In this case, students often combine the like terms first resulting in the erroneous expression 7(x + 3). If this occurs, remind students to use the Order of Operation rules; this will result in students completing the Distributive Property first.
• A common mistake occurs when students encounter problems with fractions in front of the parentheses, such as in Question 2, part (b): \( \frac{2}{3}(6x + 12) \). Students will often multiply the fraction by the first term, and in the process reduce portions of the fraction when using the dividing out process. Then, when multiplying the fraction by the second term, they erroneously continue with the reduced fraction instead of the correct initial fraction.

2. Rewrite each expression using the Distributive Property. Then, simplify if possible.
   a. \( 2(x + 4) \)
      \[ 2x + 8 \]
   b. \( \frac{3}{4}(6x + 12) \)
      \[ 4x + 8 \]
   c. \( 2(x + 5) + 4(x + 7) \)
      \[ 2x + 10 + 4x + 28 \]
      \[ 6x + 38 \]
   d. \( 5x + 2(3x - 7) \)
      \[ 5x + 6x - 14 \]
      \[ 11x - 14 \]
   e. \( 2(y + 5) + 2(x + 5) \)
      \[ 2y + 10 + 2x + 10 \]
      \[ 2x + 2y + 20 \]
   f. \( \frac{1}{2}(4x + 2) + 8x \)
      \[ 2x + 1 + 8x \]
      \[ 10x + 1 \]
Problem 3
Students represent $4x + 8$ using algebra tiles. They will divide the model into four equal groups, write an expression to represent each group, and verify the groups created are equal by multiplying the expression by 4. This process is repeated using a second expression before the Distributive Property of Division over Addition is formally introduced. A more complex problem is addressed through student work. The Distributive Property of Division over Subtraction is formally defined. The problem concludes with students simplifying expressions with varying levels of difficulty and stating what form of the Distributive Property was used.

Materials
Algebra tiles

Grouping
• Ask a student to read the information prior to Question 1 aloud. Discuss this information as a class.
• Have students complete Question 1 as a class. Then share the responses as a class.

Share Phase, Question 1
• Does it matter how you arrange the algebra tiles? Explain.
• What are equal groups?

Problem 3  Splitting an Expression Equally

So far in this lesson, you multiplied expressions together using the Distributive Property of Multiplication over Addition and the Distributive Property of Multiplication over Subtraction. Now let's think about how to divide expressions. How do you think the Distributive Property will play a part in dividing expressions? Let's find out.

1. Represent $4x + 8$ using your algebra tiles.
   a. Sketch the model you create.

   

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>1</td>
</tr>
<tr>
<td>x</td>
<td>1</td>
</tr>
<tr>
<td>x</td>
<td>1</td>
</tr>
</tbody>
</table>

   I know that multiplication and division are inverse operations. So, I should start thinking in reverse.

   

   b. Divide your algebra tile model into as many equal groups as possible. Then, sketch the model you created with your algebra tiles.

   

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>1</td>
</tr>
<tr>
<td>x</td>
<td>1</td>
</tr>
<tr>
<td>x</td>
<td>1</td>
</tr>
</tbody>
</table>

   How do you know you created equal groups?
   Does the operation of division always result in equal parts?
Grouping
Ask a student to read the information in the worked example aloud. Discuss this content as a class.

Discuss Phase, Worked Example
- Explain the mathematical notation to show the Distributive Property of Division over Addition.
- How is the process of distribution shown in the notation?
- How is the number of terms in the dividend related to the number of terms in the quotient?

Let's consider the division expression from Question 1. You can rewrite it using the Distributive Property of Division over Addition.

\[
\frac{4x + 8}{4} = \frac{4x}{4} + \frac{8}{4} = 1x + 2 = x + 2
\]

When you rewrite the expression, you have to divide the denominator into both terms in the numerator.

c. How many equal groups did you create? 4

d. Write an expression to represent each group from your sketches in part (b).
\[x + 2\]
e. Verify you created equal groups by multiplying your expression from part (d) by the number of equal groups from part (c). The product you calculate should equal \(4x + 8\).

\[4(x + 2) = 4x + 8\]
**Grouping**

Have students complete Question 2 with a partner. Then share the responses as a class.

**Share Phase, Question 2**

- Does it matter how you arrange the algebra tiles? Explain.
- Will equal groups always be able to be arranged to make a rectangle? Explain.
- How do you know you created equal groups?

The model you created in Question 1 is an example that shows that the Distributive Property holds true for division over addition.

2. Represent $2x + 6y + 4$ using your algebra tiles.
   
   a. Sketch the model you created. Be careful to not combine unlike terms using the same type of algebra tiles.
   
   $$
   \begin{array}{ccc}
   x & y & 1 \\
   x & y & 1 \\
   y & 1 \\
   y & 1 \\
   y & \\
   y & \\
   \end{array}
   $$

   b. Divide your algebra tile model into equal groups. Then, sketch the model you created with your algebra tiles.
   
   $$
   \begin{array}{ccc}
   x & 1 \\
   y & 1 \\
   y & \\
   y & \\
   y & \\
   \end{array} \quad \begin{array}{ccc}
   x & 1 \\
   y & 1 \\
   y & \\
   y & \\
   y & \\
   \end{array}
   $$

   c. How many equal groups did you create?
      
      2

   d. Write an expression to represent each group from part (b).
      
      $$x + 3y + 2$$

   e. Verify that you created equal groups by multiplying your expression from part (d) by the number of equal groups from part (c). The product should equal $2x + 6y + 4$.
      
      $$2(x + 3y + 2) = 2x + 6y + 4$$
Grouping

- Ask a student to read the definition aloud. Discuss this information and complete Question 3 as a class.
- Have students complete Question 4 with a partner. Then share the responses as a class.

Discuss Phase, Definition and Questions 3 and 4

- Explain what the algebraic statement \( \frac{a+b}{c} = \frac{a}{c} \frac{b}{c} \) means.
- Why can't \( c = 0? \)
- Does it matter how many terms are in the dividend when using the Distributive Property? Explain.
- Was it easier to simplify the expression using algebra tiles or the Distributive Property?

The Distributive Property of Division over Addition states that if \( a, b, \) and \( c \) are real numbers and \( c \neq 0 \), then \( \frac{a+b}{c} = \frac{a}{c} + \frac{b}{c} \).

3. Rewrite the division expression from Question 2 using the Distributive Property of Division over Addition. Then, simplify the expression.

\[
\frac{2x + 6y + 4}{2} = \frac{2x}{2} + \frac{6y}{2} + \frac{4}{2} = x + 3y + 2
\]

4. Represent \( 6 + 3(x + 1) \) using your algebra tiles.
   a. Sketch the model you created.
   b. Divide your algebra tile model into equal groups. Then, sketch the model you created with your algebra tiles.
   c. How many equal groups did you create?
      3
   d. Write an expression to represent each group from part (b).
      \( x + 3 \)
   e. Verify that you created equal groups by multiplying your expression from part (d) by the number of equal groups from part (c). The product you calculate should equal \( 6 + 3(x + 1) \).
      \[
      3(x + 3) = 3x + 9
      \]
      \[
      = 6 + 3x + 3
      \]
      \[
      = 6 + 3(x + 1)
      \]
Grouping

- Have students complete Question 5 with a partner. Then share the responses as a class.
- Ask a student to read the information following Question 5 aloud. Discuss the definition as a class.

Share Phase, Question 5

- What is the difference between the two methods?
- Which method do you prefer? Why?

Discuss Phase, Definition

- Explain what the algebraic statement \( \frac{a-b}{c} = \frac{a}{c} - \frac{b}{c} \) means.
- Demonstrate the use of this property by solving this problem \((18x - 30y - 24) ÷ 3\).

The expression in Question 4 can be simplified using the Distributive Property of Division over Addition in two ways.

5. Analyze each correct method. Explain the reasoning used in each.

Method 1

\[
\frac{6 + 3(x + 1)}{3} = \frac{6}{3} \frac{3(x + 1)}{3} = \frac{2 + (x + 1)}{3} = x + 3
\]

Reasoning: Method 1 applied the Distributive Property of Division over Addition, and then simplified each term.

Method 2

\[
\frac{6 + 3(x + 1)}{3} = \frac{6 + 3x + 3}{3} = \frac{3x + 9}{3} = \frac{3x}{3} + \frac{9}{3} = x + 3
\]

Reasoning: Method 2 simplified the numerator by applying the Distributive Property of Multiplication over Addition, and then combined like terms. This method then applied the Distributive Property of Division over Addition and finally simplified.

The Distributive Property also holds true for division over subtraction.

The **Distributive Property of Division over Subtraction** states that if \( a, b, \) and \( c \) are real numbers and \( c \neq 0 \), then \( \frac{a - b}{c} = \frac{a}{c} - \frac{b}{c} \).

For example: \( \frac{2x - 5}{2} = \frac{2x}{2} - \frac{5}{2} = x - \frac{5}{2} \).
Grouping

Have students complete Question 6 with a partner. Then share the responses as a class.

Share Phase, Question 6

- How many Distributive Properties are there?
- How do you know when to use which Distributive Property?
- What is the difference between the Distributive Property of Division over Addition and the Distributive Property of Division over Subtraction?

6. Simplify each expression using a Distributive Property. Then, state which Distributive Property you used.

a. \(3x - 7\)
   \[
   \frac{3x - 7}{3} = \frac{x - 7}{3}
   \]
   I used Distributive Property of Division over Subtraction to simplify the expression.

b. \(6 + 2(x + 4)\)
   \[
   \frac{6 + 2x + 8}{2} = \frac{2x + 14}{2} = x + 7
   \]
   I first used the Distributive Property of Multiplication over Addition, and then I combined like terms. Finally, I used the Distributive Property of Division over Addition.

c. \(\frac{32 + 4x}{4}\)
   \[
   \frac{32 + 4x}{4} = \frac{8 + x}{4} = x + 8
   \]
   I used the Distributive Property of Division over Addition.

d. \(\frac{2x + 7}{2}\)
   \[
   \frac{2x + 7}{2} = \frac{x + 7}{2}
   \]
   I used the Distributive Property of Division over Addition.

e. \(\frac{3(x + 1) + 12}{3}\)
   \[
   \frac{3(x + 1) + 12}{3} = \frac{(x + 1) + 4}{3} = \frac{x + 5}{3}
   \]
   I first used the Distributive Property of Division over Addition, and then I combined like terms.

Be prepared to share your solutions and methods.
Follow Up

Assignment
Use the Assignment for Lesson 8.3 in the Student Assignments book. See the Teacher's Resources and Assessments book for answers.

Skills Practice
Refer to the Skills Practice worksheet for Lesson 8.3 in the Student Assignments book for additional resources. See the Teacher's Resources and Assessments book for answers.

Assessment
See the Assessments provided in the Teacher's Resources and Assessments book for Chapter 8.

Check for Students' Understanding
Identify the property used to generate the second step in each expression.

1. \(5(3 + 7)\)
   \[15 + 35\]
   Distributive Property of Multiplication Over Addition

2. \(\frac{(24 - 8)}{8}\)
   \[24 - 8\]
   \[\text{Divide} \quad 8\]
   \[\text{Divide} \quad 8\]
   Distributive Property of Division Over Subtraction

3. \(4(9 - 7)\)
   \[36 - 28\]
   Distributive Property of Multiplication Over Subtraction

4. \(20 + (x + 6) + 7\)
   \[20 + x + (6 + 7)\]
   Associative Property of Addition

5. \(\frac{(10 + 3)}{7}\)
   \[10 + 3\]
   \[\frac{7}{7} + \frac{3}{7}\]
   Distributive Property of Division Over Addition
Essential Ideas

- Two algebraic expressions are equivalent expressions if, when any values are substituted for variables, the results are equal.
- One non-example is necessary to disprove a claim, while an infinite number of examples are necessary to prove a claim. Because it is impossible to provide an infinite number of examples, the use of properties and graphs are more effective means of proof than tables.
- Two algebraic expressions can be proven to be equivalent by: (1) using algebraic properties to simplify them until they are written the exact same way; and (2) graphing each expression on the same graph to determine that their graphs are the same.
- Tables are not an effective method to prove that two expressions are equivalent because it is impossible to substitute every possible value for the variables. Tables are an effective method to prove that two expressions are not equivalent because all that is needed is one example that the same value substituted in each expression yields different results.

Texas Essential Knowledge and Skills for Mathematics

Grade 6

(6) Expressions, equations, and relationships. The student applies mathematical process standards to use multiple representations to describe algebraic relationships. The student is expected to:

(C) represent a given situation using verbal descriptions, tables, graphs, and equations in the form \( y = kx \) or \( y = x + b \)

(7) Expressions, equations, and relationships. The student applies mathematical process standards to develop concepts of expressions and equations. The student is expected to:

(C) determine if two expressions are equivalent using concrete models, pictorial models, and algebraic representations

(D) generate equivalent expressions using the properties of operations: inverse, identity, commutative, associative, and distributive properties

Learning Goals

In this lesson, you will:
- Compare expressions using properties, tables, and graphs.
- Graph expressions on a calculator.
- Determine if two expressions are equivalent.
- Write the corresponding expressions to problem situations.

Key Terms

- equivalent expressions
Materials
Graphing Calculator

Overview
Students analyze two different expressions to represent the same situation. They use properties, tables, and graphing calculators to show that the expressions are equivalent. Students then compare other algebraic expressions and are asked to use tables and graphs to determine if they equivalent. This opens the discussion that one non-example is necessary to disprove a claim, while an infinite number of examples are necessary to prove a claim.

Students write algebraic expressions to represent different contexts, each of which can be written multiple ways, allowing the opportunity to determine if students’ expressions are equivalent or not. Number tricks are used as an additional context for students to convert mathematical words into symbols and simplify algebraic expressions. Students will use a graphic organizer to summarize the Associative, Commutative, and Distributive Properties and provide an example of each.
Match each property with the appropriate rule.

<table>
<thead>
<tr>
<th>Property</th>
<th>Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Distributive Property of Division over Subtraction (F)</td>
<td>A. ( a(b + c) = ab + ac )</td>
</tr>
<tr>
<td>2. Distributive Property of Division over Addition (C)</td>
<td>B. ( ab = ba )</td>
</tr>
<tr>
<td>3. Distributive Property of Multiplication over Subtraction (H)</td>
<td>C. ( \frac{a + b}{c} = \frac{a}{c} + \frac{b}{c} )</td>
</tr>
<tr>
<td>4. Distributive Property of Multiplication over Addition (A)</td>
<td>D. ( a + (b + c) = (a + b) + c )</td>
</tr>
<tr>
<td>5. Commutative Property of Multiplication (B)</td>
<td>E. ( ab = ba )</td>
</tr>
<tr>
<td>6. Commutative Property of Addition (G)</td>
<td>F. ( \frac{a - b}{c} = \frac{a}{c} - \frac{b}{c} )</td>
</tr>
<tr>
<td>7. Associate Property of Multiplication (E)</td>
<td>G. ( a + b = b + a )</td>
</tr>
<tr>
<td>8. Associative Property of Addition (D)</td>
<td>H. ( a(b - c) = ab - ac )</td>
</tr>
</tbody>
</table>
So, what makes a park a national park? There are different ideas on what a national park is, but many experts agree that a National Park is a government-owned plot of natural land. This land is set aside for animal safety, and people can use the land for recreation.

The first national park established was California's Yosemite (pronounced yo - SEM - ih - tee) National Park. The park exists because of two conservationists: John Muir and President Theodore Roosevelt. Both men were pioneers in land conservation. Do you think land conservation is still taking place? Do you think conservation has changed since the time of Roosevelt and Muir?
Problem 1
Students compare two expressions that represent the same situation. They will describe the meaning of each expression in terms of the problem situation, state the property that verifies the two expressions are equivalent, complete a table of values for each expression, and finally use a graphing calculator to show that the graph of each expression is the same.

Materials
Graphing calculator

Grouping
• Have students complete Question 1 with a partner. Then share the responses as a class.
• Ask a student to read the information prior to Question 2 aloud. Discuss the information and complete Question 2 as a class.

Share Phase, Question 1
• Do you need to know the total cost of the camping trip to answer the questions?
• Why do you think $p$ was used to represent the variable?
• Could a different letter be used to represent the variable?
• What does the variable represent in the problem situation?
• What does the number 4 represent in the problem situation?

Discuss Phase, Question 2
• How can you tell if two expressions are equivalent?
• What does it mean to say that two expressions are equivalent?
• What methods can be used to show that two expressions are equivalent?
• What properties can be used to show that two expressions are equivalent?
Grouping
Have students complete Question 3 with a partner. Then share the responses as a class.

**Share Phase, Question 3**
- What do you notice about the second and third columns in the table?
- Using a table of values, how can you tell the two expressions are equivalent?
- What would the table look like if the expressions were not equivalent?
- How many matching entries must you use in a table to claim that the two expressions are equivalent?
- How many non-matching entries must you use in a table to determine that the expressions are not equivalent?

3. Use the table to answer the questions.
   a. Complete the table shown for each additional number of tins sold.

<table>
<thead>
<tr>
<th>Number of Additional Popcorn Tins Sold</th>
<th>Harley’s Expression 4(p + 5)</th>
<th>Jerome’s Expression 4p + 20</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>32</td>
<td>32</td>
</tr>
<tr>
<td>7</td>
<td>48</td>
<td>48</td>
</tr>
<tr>
<td>15</td>
<td>80</td>
<td>80</td>
</tr>
<tr>
<td>25</td>
<td>120</td>
<td>120</td>
</tr>
</tbody>
</table>

b. What can you determine based on the values in the table?
   It doesn’t matter what value of \( p \) is substituted into each expression, the result for both Harley’s expression and Jerome’s expression is the same.

c. What would you need to do to verify the two expressions are equivalent?
   I would have to calculate the value of each expression for all possible values of the variable.
Grouping

- Ask a student to read the worked example aloud. Discuss the information as a class.
- Have students complete Questions 4 and 5 with a partner. Then share the responses as a class.

Share Phase, Questions 4 and 5

- Where is the Y= key on your calculator?
- Where is the WINDOW key on your calculator?
- Where is the GRAPH key on your calculator?
- Where is the parenthesis key on your calculator?
- What happens if you forget to enter the parenthesis?
- If you do not change the WINDOW, what does the graph look like?
- How do you know when you need to change the WINDOW?
- What does Xmin mean?
- What does Xmax mean?
- What affect does changing the Xscl or the Yscl do to the graph?
- Why do you only see one graph if you entered two equations?
- Does the graph of the expression give you any more information than the table of values?
- How can you determine more values on the graph?

You can use a graphing calculator to determine if two expressions are equivalent. The graphing calculator will create a graph for each expression. Follow the steps.

**Step 1:** Press \[ \text{Y=} \] and enter what is shown.

\[
y_1 = 4(x + 5) \\
y_2 = 4x + 20
\]

To distinguish between the graphs of \(y_1 \) and \(y_2\), move your cursor to the left of \(y_1\) until the \(\backslash\) flashes. Press \[ \text{ENTER} \] one time to select \(\backslash\).

**Step 2:** Press \[ \text{WINDOW} \] to set the bounds and intervals for the graph.

\[
X_{\text{min}} = 0, X_{\text{max}} = 50, \text{ and } X_{\text{scl}} = 10, \\
Y_{\text{min}} = 0, Y_{\text{max}} = 100, \text{ and } Y_{\text{scl}} = 10.
\]

**Step 3:** Press \[ \text{GRAPH} \].

4. Sketch both graphs on the coordinate plane shown.

5. How does the graph verify that the two expressions are equivalent? The graphs are the same line.

- What would the graph look like if the expressions were not equivalent?
- Do the graphs have to be lines to be equivalent?
- What methods are efficient methods to prove two expressions are equivalent: the use of properties, tables and/or graphs? Explain.
Problem 2
Students determine if two expressions are equivalent using tables and graphs. This problem also addresses the concept of proof; basically what is enough to prove a claim and what is enough to disprove a claim.

Materials
Graphing calculator

Grouping
Have students complete Questions 1 and 2 with a partner. Then share the responses as a class.

ELL Tip
Allow English Language Learners to engage in a free discussion expressing their ideas for determining equivalence using the W.I.T. Questioning model:
• Why do you think Question 1 isn’t, “Are the expressions equivalent?”
• Is there a way to tell if two expressions are equivalent using a table?
• Tell me more about determining equivalence and non-equivalence.
Listen for an understanding that verifying equivalence requires calculating each expression for all possible values. Determining non-equivalence takes just one counter-example.
Share Phase, Question 1

- What advantages are there to using a table of values to determine if two expressions are equivalent?

- What advantages are there to using a graphing calculator to determine if two expressions are equivalent?

- Is it easier to use a table of values or a graphing calculator to determine if two expressions are equivalent?

- What gives you more information about the situation, a table of values or the graph?

- How can you tell if your answer will be a fraction or not prior to calculating its value?

- Monica said she can tell if two expressions are equivalent or not after substituting one value of \( x \) into each expression. Is she correct? Explain.

Share Phase, Question 2

- How do you enter a fraction on the graphing calculator?

- What are the bounds on a graphing calculator?

- How do you know when you need to change the bounds?

- Use properties to show whether the two expressions are equivalent or not.

---

2. Determine if the two expressions are equivalent. Graph each expression using a graphing calculator and sketch the graph in the coordinate plane shown. Press [WINDOW] to set the bounds: \( \text{Xmin} = 0, \text{Xmax} = 10, \text{Xscl} = 1, \text{Ymin} = 0, \text{Ymax} = 10, \) and \( \text{Yscl} = 1. \)

   a. Are the two expressions equivalent? Explain your reasoning.

   \[
   2(x + 2) + 3x \\
   5x + 4
   \]

   The two expressions are equivalent. They graph the same line.

   b. Are the two expressions equivalent?

   Explain your reasoning.

   \[
   \frac{1}{2}x + 5 \\
   \frac{1}{2}(x + 5)
   \]

   The two expressions are not equivalent. The graphs are of two separate lines.
Problem 3
Students write algebraic expressions to represent four different situations. Each expression can be written multiple ways, one way involving the use of the Distributive Property.

Grouping
Have students complete Questions 1 through 4 with a partner. Then share the responses as a class.

Share Phase, Question 1
- Does each cook prepare the same number of meals?
- Why was \( m \) chosen to represent the variable? What does it represent in this problem situation?
- What does the number 21 represent in the problem situation? Why is it added when creating the expression?
- What property can be associated with this problem situation?
- Explain how your algebraic expression relates to the problem situation.
- Can the algebraic expression be written in another way? Will the expressions be equivalent?

Share Phase, Question 2
- Why was \( c \) chosen to represent the variable? What does it represent in this problem situation?

Problem 3 Writing Expressions

1. A local restaurant is busiest over lunch and has three cooks who work at this time. The cooks divide the incoming orders among themselves. So far today, the cooks have prepared 21 meals total.
   a. If 18 additional orders come in, how many meals will each cook prepare? Write an expression, and show your calculations.
      \[
      \frac{21 + 18}{3} = 13
      \]
      Each cook will prepare 13 meals.
   b. If 42 additional orders come in, how many meals will each cook prepare? Write an expression, and show your calculations.
      \[
      \frac{21 + 42}{3} = 21
      \]
      Each cook will prepare 21 meals.
   c. Write an expression to represent the unknown number of meals each cook prepares. Let \( m \) represent the number of additional orders.
      \[
      \frac{21 + m}{3} = 7 + \frac{1}{3}m
      \]

2. Christopher is selling oatmeal energy bars to raise money for his baseball team. The team receives $1.25 for each oatmeal energy bar sold. He has already sold 25 oatmeal energy bars.
   a. If Christopher sells 10 more energy bars, how much money will he raise for the baseball team? Write an expression, and show your calculations.
      \[
      1.25(25 + 10) = 43.75
      \]
      He will raise $43.75 for his baseball team.
   b. If Christopher sells 45 more energy bars, how much money will he raise for the baseball team? Write an expression, and show your calculations.
      \[
      1.25(25 + 45) = 87.50
      \]
      He will raise $87.50 for his baseball team.
   c. Write an expression to represent the unknown amount of money Christopher will raise for his baseball team. Let \( c \) represent the number of additional energy bars sold.
      \[
      1.25(25 + c) = 31.25 + 1.25c
      \]
   - What does the number 25 represent in the problem situation? Why is it added when creating the expression?
   - What property can be associated with this problem situation?
   - Explain how your algebraic expression relates to the problem situation.
   - Can the algebraic expression be written in another way? Will the expressions be equivalent?
Share Phase, Question 3

- Is joining the club really at no cost? How does the club make its money?
- What happens if you do not purchase 12 more CDs within the first year?
- Is this a good deal?
- Why was \( m \) chosen to represent the variable? What does it represent in this problem situation?
- Why is the number 10 subtracted when creating the expression?
- What property can be associated with this problem situation?
- Explain how your algebraic expression relates to the problem situation.
- Can the algebraic expression be written in another way? Will the expressions be equivalent?

Share Phase, Question 4

- What does profit mean? Do all the friends equally share the profits?
- What does \$100 represent in the problem situation?
- What does the phrase “break even” mean? What is the break-even point?
- Why do you need to use a fraction when creating the expression?
- Why was \( d \) chosen to represent the variable? What does it represent in this problem situation?

3. The Music-For-All Club is offering a special deal to all new members. You can join the club at no cost and receive 10 free CDs. However, you must also agree to purchase at least 12 more CDs within the first year at the club’s special price of \$11.99 per CD.
   a. What will 20 CDs from the club cost? Show your calculations.
      \[
      11.99(20 - 10) = 119.90 \\
      \text{Twenty CDs will cost \$119.90.}
      \]
   b. What will 25 CDs from the club cost? Show your calculations.
      \[
      11.99(25 - 10) = 179.85 \\
      \text{Twenty-five CDs will cost \$179.85.}
      \]
   c. Write an expression to represent the total cost of an unknown number of CDs.
      Let \( m \) represent the number of CDs you will get from the club.
      \[
      11.99(m - 10) = 11.99m - 119.90
      \]

4. Four friends decide to start a summer business doing yard work in their neighborhood. They will split all their earnings evenly. They have two lawnmowers, but they need to buy gas, rakes, trash bags, and a pair of pruners. They spend \$100 buying supplies to get started.
   a. How much profit will each friend receive if they earn \$350 the first week? Show your calculations.
      \[
      \frac{1}{4}(350 - 100) = 62.50 \\
      \text{Each friend will receive \$62.50.}
      \]
   b. How much profit will each friend receive if they earn \$475 the first week? Show your calculations.
      \[
      \frac{1}{4}(475 - 100) = 93.75 \\
      \text{Each friend will receive \$93.75.}
      \]
   c. Write an expression that represents the unknown profit for each friend.
      Let \( d \) represent the amount of money earned.
      \[
      \frac{1}{4}(d - 100) = \frac{1}{4}d - 25
      \]
Problem 4
A number trick is used to illustrate the power of mathematics. Students pick a number between 1 and 30 and follow steps to conclude that the number they chose had no affect on the final resulting number, in fact, everyone had the same result. They will use the properties of real numbers to unravel the mystery, letting \( n \) represent the original number. The Check for Understanding activity in this lesson asks students to create their own number trick.

Grouping
Have students complete Questions 1 through 5 with a partner. Then share the responses as a class.

Share Phase, Questions 1 through 5
- What properties can be associated with this number trick?
- Could you use this trick to design your own number trick?
- By reading the steps in the trick, can you identify which number properties were used?
- Can you design a trick that would result in any number you choose?

Problem 4 Number Magic

1. Try this number riddle with a partner. Follow the steps.
   - Step 1: Pick a number between 1 and 30.
   - Step 2: Add 9 to your number.
   - Step 3: Multiply the sum by 3.
   - Step 4: Subtract 6 from the product.
   - Step 5: Divide the difference by 3.
   - Step 6: Subtract your original number.

2. What is your answer? Compare your answer to your partner's answer. What do you notice?
   - We both got the number 7.

3. Why do you think you will always end with the same number?
   - Answers will vary.

Let's use the properties of real numbers to investigate why this riddle works.

4. What quantity is changing in this riddle?
   - The original number changes with each step.

5. Let \( n \) represent the original number. Write and simplify an expression for each step.
   - Step 1: \( n \)
   - Step 2: \( n + 9 \)
   - Step 3: \( 3(n + 9) = 3n + 27 \)
   - Step 4: \( 3n + 27 - 6 = 3n + 21 \)
   - Step 5: \( \frac{3n + 21}{3} = n + 7 \)
   - Step 6: \( (n + 7) - n = 7 \)
Talk the Talk

Students use the graphic organizer to summarize the Associative, Commutative, and Distributive properties and provide an example of each.

Grouping

Have students complete the graphic organizer independently. Then share the responses as a class.

Share Phase, Talk the Talk

- How can you tell the difference between the Commutative property and the Associative property?
- The order of operation rules state to simplify expressions within parentheses first. Explain how the Distributive property relates to the order of operations rule involving parentheses.

**Talk the Talk**

Complete the graphic organizers by stating each rule using variables and providing an example.

**CONMUTATIVE PROPERTY OF ADDITION**

For any number $a$ and $b$

$a + b = b + a$

Example:

$x + 4 = 4 + x$

$6 = 6$

**CONMUTATIVE PROPERTY OF MULTIPLICATION**

For any number $a$ and $b$

$ab = ba$

Example:

$4 \times 3 = 3 \times 4$

$12 = 12$

**ASSOCIATIVE PROPERTY OF ADDITION**

For any number $a$, $b$, and $c$

$(a + b) + c = (a + b) + c$

Example:

$(4 + 3) + 2 = 4 + (3 + 2)$

$12 + 2 = 4 + 10$

$14 = 14$

**ASSOCIATIVE PROPERTY OF MULTIPLICATION**

For any number $a$, $b$, and $c$

$a(bc) = (ab)c$

Example:

$(2(4 \times 5)) = (2 \times 4)5$

$10 \times 20 = 80$

$200 = 200$
8.4 Multiple Representations of Equivalent Expressions

**Multiplication over Addition**
For any number $a$, $b$, and $c$

$$a(b + c) = ab + ac.$$  

Example:

$$5(20 + 7) = 5 \times 20 + 5 \times 7$$
$$5(27) = 100 + 35$$
$$135 = 135$$

**Multiplication over Subtraction**
For any number $a$, $b$, and $c$

$$a(b - c) = ab - ac.$$  

Example:

$$5(3 - 2) = 5 \times 3 - 5 \times 2$$
$$5(1) = 15 - 10$$
$$5 = 5$$

**Division over Addition**
For any number $a$, $b$, and $c$

$$\frac{(a + b)}{c} = \frac{a}{c} + \frac{b}{c}.$$  

Example:

$$\frac{(8 + 4)}{2} = \frac{8}{2} + \frac{4}{2}$$
$$4 + 2 = 6$$

**Division over Subtraction**
For any number $a$, $b$, and $c$

$$\frac{(a - b)}{c} = \frac{a}{c} - \frac{b}{c}.$$  

Example:

$$\frac{(8 - 4)}{2} = \frac{8}{2} - \frac{4}{2}$$
$$4 - 2 = 2$$

Be prepared to share your solutions and methods.
Follow Up

Assignment
Use the Assignment for Lesson 8.4 in the Student Assignments book. See the Teacher’s Resources and Assessments book for answers.

Skills Practice
Refer to the Skills Practice worksheet for Lesson 8.4 in the Student Assignments book for additional resources. See the Teacher’s Resources and Assessments book for answers.

Assessment
See the Assessments provided in the Teacher’s Resources and Assessments book for Chapter 8.

Check for Students’ Understanding
1. Use algebra to show why this number trick works.
   • Choose a Number $x$
   • Add 5 $x + 5$
   • Double the result $2(x + 5) = 2x + 10$
   • Subtract 4 $2x + 10 - 4 = 2x + 6$
   • Divide the result by 2 $\frac{2x + 6}{2} = x + 3$
   • Subtract the number you started with $x - x + 3 = 3$
   • The result is 3

2. Use algebra to create your own number trick. Show why it works.
   • Choose a Number $x$
   • Add 3 $x + 3$
   • Double the result $2(x + 3) = 2x + 6$
   • Add 4 $2x + 6 + 4 = 2x + 10$
   • Divide the result by 2 $\frac{2x + 10}{2} = x + 5$
   • Subtract the number you started with $x - x + 5 = 5$
   • The result is 5
Essential Ideas

- Algebra tiles can be used to model algebraic expressions. The tiles can then be rearranged so that multiple terms with identical tiles, called like terms, can be combined into one term.
- The process of rearranging algebra tiles connects to the commutative, associative, and distributive properties.
- For algebraic expressions that require multiple operations to simplify, the use of the distributive property should be completed prior to combining like terms. The algebraic expression is simplified completely once all like terms are combined.

Texas Essential Knowledge and Skills for Mathematics

Grade 6

(7) Expressions, equations, and relationships. The student applies mathematical process standards to develop concepts of expressions and equations. The student is expected to:

(D) generate equivalent expressions using the properties of operations: inverse, identity, commutative, associative, and distributive properties

Materials

Algebra tiles
Overview
Students use algebra tiles to model algebraic expressions. The models are used to combine like terms, and this process is then connected to the use of mathematical properties. Students then simplify algebraic expressions using of the Distributive property and combining like terms without using algebra tiles.
Warm Up

Are the given expressions equivalent? Show your work. Describe the method used.

1. \(6(x - 4)\) \(6x - 4\)

Work and descriptions may vary. One possible response is:

\(6x - 24\) \(6x - 4\)

These two expressions are not equivalent. I used the distributive property of multiplication over subtraction to simplify the first expression and saw that it did not match the second expression.

2. \(\frac{15x + 10}{5}\) \(3x + 2\)

Work and descriptions may vary. One possible response is:

\(3x + 2\) \(3x + 2\)

These two expressions are equivalent. I used the distributive property of division over addition to simplify the first expression and saw that it matched the second expression.

3. What other methods could you have used to show whether the expressions are equivalent or not?

For Question 1, I could have used tables or graphs to show that the expressions were not equivalent. For Question 2, I could have used a graph to show that the expressions were equivalent; a table of values would not be enough proof to show that the expressions were equivalent.
Using categories to classify people, places, and other objects is often very useful. For example, the population of the United States is sometimes divided into rural, urban, and suburban. What does each of the categories mean and why are they useful? What are some other categories that are used to describe the population in the United States?
Problem 1

The dimensions of algebra tiles are reviewed. Students use algebra tiles to model algebraic expressions. The models are then used to make sense of combining like terms. The problem concludes with students creating and solving their own problem that requires combining like terms.

Materials
Algebra tiles

Grouping
Ask a student to read the introduction to Problem 1 aloud. Discuss the information and complete Question 1 as a class.

Discuss Phase, Question 1
- What do the values written inside each square represent?
- Explain why the smallest tile can be called a unit square.
- What tiles have a side length in common with the unit square? Explain.
- How can I combine an $x$-tile, a $y$-tile and one unit square to make one large rectangle? What are the dimensions of this new rectangle?
- What tiles have a side length in common with the larger squares? Explain.
- How can I combine an $x^2$-tile and an $x$-tile to make one large rectangle? What are the dimensions of this new rectangle?

Problem 1 Like and Unlike

You will use the algebra tiles shown to model algebraic expressions.

1. What are the dimensions of each of the tiles?
   a. This tile is one unit by $x$ units.
   b. This tile is one unit by $y$ units.
   c. This tile is one unit by one unit.
   d. This tile is $y$ units by $y$ units.
   e. This tile is $x$ units by $x$ units.
Grouping
Have students complete Questions 2 and 3 with a partner. Then share the responses as a class.

Share Phase, Questions 2 and 3
- For Question 2, if more than one tile is needed to represent a term, does it matter if those tiles are connected to make one large rectangle or are placed with spaces between them?
- What properties allow you to move the tiles and regroup them with similar tiles together?
- What are like terms? How are like terms represented with the algebra tiles?
- Create a model with all like terms.
- Create a model with no like terms.
- Compare the algebraic expression with the original expression in Question 2. What do you notice?

2. Represent each algebraic expression using algebra tiles and the operations symbols. Then sketch your model.
   a. $3x^2 + x + 2$
   b. $2 + x^2 + y^2 + 1$

The first expression has 3 terms and 2 operations so I will have 3 groups of algebra tiles separated by 2 addition signs.
3. Which of the expressions in parts (a)–(e) have different terms with identical tiles?

In Questions 2b, 2c, 2d, and 2e, the expressions all have more than one term with identical tiles.
Grouping
Have students complete Questions 4 through 7 with a partner. Then share the responses as a class.

4. Rearrange the tiles in Question 2, part (c) so that there are only two terms.
   a. Sketch your new model. Include the operation symbol.
      
      \[
      \begin{array}{c}
      x \\
      x \\
      x \\
      x \\
      + \\
      y \\
      y \\
      \end{array}
      \]
   
   b. Write the algebraic expression represented.
      \[4x + 2y\]
   
   c. Could you rearrange the tiles so there was only one term? Explain your reasoning.
      No. I cannot combine all the tiles using only one term because the tiles for \(x\) and \(y\) are different.
   
   d. What are the terms that can be combined called?
      They are called like terms.

5. Rearrange the tiles in Question 2 part (b) to reduce the number of terms to the fewest possible.
   a. Sketch your new model. Include the operation symbol.
      
      \[
      \begin{array}{c}
      x^2 \\
      + \\
      y^2 \\
      + \\
      1 \\
      1 \\
      1 \\
      \end{array}
      \]
   
   b. Write the algebraic expression represented.
      \[x^2 + y^2 + 3\]
   
   c. Could you rearrange the tiles so there was only one term? Explain your reasoning.
      No. I cannot combine all the tiles using only one term because the tiles are different.
6. Rearrange the tiles in Question 2 part (d) to reduce the number of terms to the fewest possible.
   a. Sketch your new model. Include the operation symbol.
      \[ \text{\begin{array}{c}
      x \\
      x \\
      x \\
      x \\
      x \\
      1 \\
      \end{array}} + \text{\begin{array}{c}
      1 \\
      1 \\
      1 \\
      \end{array}} + \text{\begin{array}{c}
      y^2 \\
      \end{array}} \]
   b. Write the algebraic expression represented.
      \[ 5x + y^2 + 4 \]
   c. Could you rearrange the tiles so there are fewer terms? Explain your reasoning.
      No. I cannot rearrange the tiles so that there are fewer terms, because the tiles are different.

7. Rearrange the tiles in Question 2 part (e) to reduce the number of terms to the fewest possible.
   a. Sketch your new model. Include the operation symbol.
      \[ \text{\begin{array}{c}
      x \\
      x \\
      x \\
      x \\
      x \\
      \end{array}} + \text{\begin{array}{c}
      1 \\
      1 \\
      1 \\
      \end{array}} + \text{\begin{array}{c}
      x^2 \\
      \end{array}} \]
   b. Write the algebraic expression represented.
      \[ 5x + x^2 + 3 \]
   c. Could you rearrange the tiles so there are fewer terms? Explain your reasoning.
      No. I cannot rearrange the tiles so that there are fewer terms, because the tiles are different.
Grouping
Have students compete Questions 8 and 9 with a partner. Then share the responses as a class.

Share Phase, Question 8
• In what ways is this expression and algebra tile model different from the previous ones?
• How is the distributive property modeled using the algebra tiles?
• How is subtraction modeled using the algebra tiles?
• Compare the algebraic expression in Question 8, part (c) with the original expression and response to Question 8, part (a). What do you notice?

8. The algebraic expression $2(3x + 2) - 3$ is represented with algebra tiles and the operations symbols.

a. Use the distributive property to rewrite the expression without parentheses.

$$2(3x + 2) - 3 = 6x + 4 - 3$$

b. Model your new expression using algebra tiles and operations symbols.

$$6x + 1$$

c. Rearrange the tiles to reduce the number of terms to the fewest possible terms. Then, write the algebraic expression that is represented.
Share Phase, Question 9

For Question 9, why does everyone’s final algebraic expression contain two terms?

9. Now, you will create your own expression and model.
   a. Write your own algebraic expression that has 4 terms with 2 pairs of like terms.
      For example, \(2x + y^2 + 3x + 2y^2\)
   b. Model your expression using algebra tiles and operations symbols.
      \[\text{tiles for } 2x + y^2 + 3x + 2y^2\]
   c. Rearrange the tiles to reduce the number of terms to the fewest possible.
      \[\text{rearranged tiles for } 5x + 3y^2\]
   d. Write the algebraic expression that is represented.
      \(5x + 3y^2\)
   e. Could you rearrange the tiles so there are fewer terms? Explain your reasoning.
      No. I cannot rearrange the tiles so that there are fewer terms, because the tiles are different.
Problem 2
Students simplify algebraic expressions by combining like terms using algebra tiles. Equivalent algebraic expressions and equivalent models are made explicit to students. Students will connect the algebra tile models with the mathematical properties that they have learned.

Materials
Algebra tiles

Grouping
Have students complete Questions 1 through 3 with a partner. Then share the responses as a class.

Share Phase, Question 1
- Were you able to write the algebraic expression without moving the tiles? If so, explain how you solved the problem in your mind.
- Why do the two tile arrangements look different?
- How do the two tile arrangements relate to Question 1, part (d)?
- What equivalencies do you see in this question?

Problem 2  What about the Expressions?
1. Represent $4x + 2x$ using algebra tiles and the operations symbols.
   a. Sketch the model of the expression.
      
      ![Sketch of algebra tiles]

   b. Write an equivalent algebraic expression with one term.
      
      $6x$

   c. Represent the algebraic expression from part (b) using algebra tiles and operators.
      
      ![Represented algebra expression]

   d. What property tells you that $4x + 2x = (4 + 2)x$?
      
      The Distributive Property of Multiplication over Addition
Share Phase, Question 2

- For Question 2, part (d), what would you have to do with the model of $2x + 5 + 3x$ to create the model of $2x + 3x + 5$?
- Is the associative property used anywhere in this question? If so, how?
- How is the symbolic representation of the associative property different than the symbolic representation of the distributive property?
- How could you have solved Question 2, part (b) without using algebra tiles?

2. Represent $2x + 5 + 3x$ using algebra tiles and the operations symbols.
   a. Sketch the model of the expression.
      
      ![Algebra Tiles Model]

   b. Write an equivalent algebraic expression by combining like terms.
      $5x + 5$

   c. Represent the algebraic expression from part (b) using algebra tiles and operations symbols.
      
      ![Algebra Tiles Model]

   d. What property tells you that $2x + 5 + 3x = 2x + 3x + 5$?
      The Commutative Property of Addition

   e. What property tells you that $2x + 3x + 5 = (2 + 3)x + 5$?
      The Distributive Property of Multiplication over Addition
Share Phase, Question 3

How could you have solved Question 3, part (b) without using algebra tiles?

3. Represent $7x + 2x^2 - 3x - x^2$ using algebra tiles and operations symbols.
   a. Sketch the model of the expression.
   
   
   b. Write an equivalent algebraic expression by combining like terms.
   
   $4x + x^2$
   
   c. Represent the algebraic expression from part (b) using algebra tiles and operations symbols.

The tiles really help me see what is like and unlike!
Problem 3
Students simplify algebraic expressions by combining like terms or by both using the Distributive Property and combining like terms without using algebra tiles.

Grouping
Have students complete Questions 1 through 12 with a partner. Then share the responses as a class.

Note
Allow struggling students to use the algebra tiles until they feel comfortable solving these problems without them.

Misconception
Outside of the mathematics classroom, combine generally means to add. It may need to be made explicit to students that combining like terms means to add or subtract based upon the signs in the problem.

Share Phase, Problem 3
- Did you rewrite the problem with like terms placed together to solve this problem?
- If so, how did you know what sign to place between the terms when you moved them?
- If not, explain how you knew what terms were to be combined and how you knew whether to add or subtract them.

Problem 3 Without the Tiles
Simplify each algebraic expression by combining like terms.

1. \(3x + 5y - 3x + 2y = 7y\)

2. \(4x^2 + 4y + 3x + 2y^2 = 4x^2 + 4y + 3x + 2y^2\)

3. \(7x + 5 - 6x + 2 = 7x - 6x + 5 + 2 = x + 7\)

4. \(x^2 + 5y + 4x^2 - 3y = x^2 + 4x^2 + 5y - 3y = 5x^2 + 2y\)

5. \(4x^3 - 5y + 3x^2 + 2x^2 = 4x^3 + 3x^2 + 2x^2 - 5y = 4x^3 + 5x^2 - 5y\)

6. \(x + 5y + 6x - 2y - 3x + 2y^2 - 6x - 3x + x + 5y - 2y + 2y^2 = 4x + 3y + 2y^2\)

Simplify each algebraic expression using the distributive properties first, and then combining like terms.

7. \(4(x + 5y) - 3x = 4x + 20y - 3x = x + 20y\)

8. \(2(2x + 5y) + 3(x + 3y) = 4x + 10y + 3x + 9y = 7x + 19y\)

9. \(3x + 5(2x + 7) = 3x + 10x + 35 = 13x + 35\)

10. \(\frac{4x + 6y}{2} - 3y = 2x + 3y - 3y = 2x\)

11. \(3(x + 2y) + \frac{3x - 9y}{3} = 3x + 6y + x - 3y = 4x + 3y\)

12. \(2(x + 3y) + 4(x + 5y) - 3x = 2x + 6y + 4x + 20y - 3x = 3x + 26y\)

Be prepared to share your solutions and methods.

- How do you know when your algebraic expression is simplified as much as possible?
- What property explains why both answers are equivalent?
- What is the coefficient of \(x\)? What is the coefficient of \(-x\)?
- If a problem requires both the use the Distributive Property and combining like terms, does it matter what operation is completed first? Why or why not.
Follow Up

Assignment
Use the Assignment for Lesson 8.5 in the Student Assignments book. See the Teacher’s Resources and Assessments book for answers.

Skills Practice
Refer to the Skills Practice worksheet for Lesson 8.5 in the Student Assignments book for additional resources. See the Teacher’s Resources and Assessments book for answers.

Assessment
See the Assessments provided in the Teacher’s Resources and Assessments book for Chapter 8.

Check for Students’ Understanding
Directions:
1. Mark each solved problem as right or wrong.
2. Circle each mistake.
3. Next to each wrong answer, redo the problem correctly.

1. \(4x - x^2 + 3x\)  
   \(4x \bigcirc 3x \bigcirc x^2\)  
   \(x + x^2\)  
   \(4x + 3x - x^2\)  
   \(7x - x^2\)  
   Wrong

2. \(3 + 2(x + 6y)\)  
   \(5(x + 6y)\)  
   \(5x + 30y\)  
   \(3 + 2x + 12y\)  
   Wrong

3. \(7x + \frac{12x - 8}{4} + 5x\)  
   \(7x + 3x - 2 + 5x\)  
   \(10x - 2 + 5x\)  
   \(15x - 2\)  
   Right

4. \(4(8y^2 + 1) + 3(2y^2 + 7)\)  
   \(32y^2 + 1\bigcirc 6y^2 \bigcirc 7\)  
   \(38y^2 + 8\)  
   \(32y^2 + 4 + 6y^2 + 21\)  
   \(38y^2 + 25\)  
   Wrong
Essential Ideas

- Many real-life situations can be represented using algebraic expressions. The algebraic expressions can then be used to answer questions about the situation.
- Different algebraic expressions may represent the same real-life situation depending upon what the initial variable represents.

Texas Essential Knowledge and Skills for Mathematics

Grade 6

(7) Expressions, equations, and relationships. The student applies mathematical process standards to develop concepts of expressions and equations. The student is expected to:

(D) generate equivalent expressions using the properties of operations: inverse, identity, commutative, associative, and distributive properties
Overview

In the first problem, students write four different sets of algebraic expressions to represent the same situation, each time basing their expressions upon a different initial variable representing a different varying quantity in the problem. Students will use the algebraic expressions to answer other questions about the same situation.

The second problem is set up the same as the first problem, the only differences being the context, the number of relationships described in the problem, a minor change among the way the relationships are described, and the number of algebraic expressions students are asked to write.
Warm Up

1. Blake is twice as old as Alec.
   a. If Alec is 10 years old, how old is Blake?
      Blake is 20 years old.
   b. If Blake is 10 years old, how old is Alec?
      Alec is 5 years old.

2. Blake is twice as old as Alec.
   Celia is 3 years older than Blake.
   a. If Alec is 9 years old, how old is Blake?
      Blake is 18 years old.
   b. If Alec is 9 years old, how old is Celia?
      Celia is 21 years old.
   c. If Celia is 13 years old, how old is Blake?
      Blake is 10 years old.
   d. If Celia is 13 years old, how old is Alec?
      Alec is 5 years old.
   e. If Blake is 30 years old, how old is Alec?
      Alec is 15 years old.
   f. If Blake is 30 years old, how old is Celia?
      Celia is 33 years old.
Learning Goals
In this lesson, you will:

- Represent problem situations with algebraic expressions.
- Analyze and solve problems with algebraic expressions.

People just love collecting souvenirs. From theaters to zoos, from amusement parks to concert halls, souvenirs are a staple of most public events and locations. Film souvenirs are very popular among movie fans. Collectors will try to get their hands on souvenirs from a film.

One of the most popular things movie fans collect are the movie posters—the ones like you see at the movie theater. Collectors will often get a copy and frame it—and sometimes try to get the autographs of the director, crew, and actors. Another collectible might be a copy of the screenplay—the actual script of a movie—or a director’s chair. Do you think music fans or art fans collect souvenirs too?
Problem 1
A context is provided that relates the number of DVDs each of four different students own based upon the number of DVDs other students in the group own. Students write four different sets of algebraic expressions to represent the situation, each time basing their expressions upon a different initial variable that represents the number of DVDs a different student owns. Students then use the algebraic expressions to answer other questions about the same situation.

Grouping
Have students complete Questions 1 through 3 with a partner. Then share the responses as a class.

Share Phase, Questions 1 and 2
- Who has the least number of DVDs? How can you tell?
- Who has the most DVDs? How can you tell?
- If you know how many DVDs Jack has, could you calculate how many DVDs everyone else has? Explain your reasoning.
- If you know how many DVDs John has, could you calculate how many DVDs everyone else has? Explain your reasoning.
- If John has 20 DVDs, how many DVDs does Jeannie have?

Problem 1  DVDs
Problem 1 DVDs

Jack, Jenny, John, and Jeannie each collect DVDs. Jack likes western movies, Jenny likes comedies, John likes action movies, and Jeannie likes sports movies.

Jenny says: “I have twice as many DVDs as Jack.”
John says: “I have four more DVDs than Jenny.”
Jeannie says: “I have three times as many as John.”

1. If Jack has 10 DVDs, determine the number of DVDs for each friend. Explain your reasoning.
   a. Jenny
      Jenny has 20 DVDs, which is twice as many as Jack.
   b. John
      John has 24 DVDs, which is four more than Jenny has.
   c. Jeannie
      Jeannie has 72 DVDs, which is three times as many as John.
   d. all of the four friends together
      Altogether they would have 126 DVDs.

2. If Jeannie has 24 DVDs, determine the number of DVDs for each friend. Explain your reasoning.
   a. John
      John has 8 DVDs because Jeannie has three times as many.
   b. Jenny
      Jenny has 4 DVDs because John has four more than Jenny.
   c. Jack
      Jack has 2 DVDs because Jenny has twice as many as Jack.
   d. all of the four friends together
      Altogether they would have 38 DVDs.

- If John has 20 DVDs, how many DVDs does Jenny have?
- If John has 20 DVDs, how many DVDs does Jack have?
- If you know how many DVDs John has, is it easiest to calculate the number of DVDs Jeannie, Jenny or Jack has? Explain.
- Do you think it is it easier to calculate the number of DVDs each person has by starting with the number of DVDs Jack has or the number of DVDs Jeannie has? Explain your reasoning.
Share Phase, Question 3

- Why are parentheses needed in Question 3, part (c)?
- How could you check that your expressions are correct for Question 3, parts (a), (b), and (c)?

Grouping

Have students complete Questions 4 and 5 with a partner. Then share the responses as a class.

Share Phase, Questions 4 and 5

- Compare your answers from Question 4, parts (a) and (b) with the answers to Questions 1, part (d) and 2, part (d). What do you notice?
- What other method could you use to solve Question 4 without using your algebraic expression?
- Why was it helpful to use the algebraic expression for Question 4?

3. Let \( x \) represent the number of DVDs that Jack has. Write an algebraic expression that represents the number of DVDs for each friend.
   a. Jenny
      Jenny has \( 2x \) DVDs.
   b. John
      John has \( 2x + 4 \) DVDs.
   c. Jeannie
      Jeannie has \( 3(2x + 4) \) or \( 6x + 12 \) DVDs.
   d. all four friends together
      They have \( x + 2x + 2x + 4 + 3(2x + 4) \) DVDs.
   e. Simplify the expression you wrote in part (d).
      \( 11x + 16 \)

4. Use your expression from part (e) to determine the number of DVDs they have altogether if Jack has:
   a. 10 DVDs.
      \[ 11(10) + 16 = 110 + 16 = 126 \]
      They have 126 DVDs.
   b. 2 DVDs.
      \[ 11(2) + 16 = 22 + 16 = 38 \]
      They have 38 DVDs.
   c. 25 DVDs.
      \[ 11(25) + 16 = 275 + 16 = 291 \]
      They have 291 DVDs.
   d. 101 DVDs.
      \[ 11(101) + 16 = 1111 + 16 = 1127 \]
      They have 1127 DVDs.

5. Write an algebraic expression to represent the number of DVDs for:
   a. the males.
      The males have \( 3x + 4 \).
   b. the females.
      The females have \( 8x + 12 \).
Grouping

- Have students complete Question 6 with a partner. Then share the responses as a class.
- Have students complete Questions 7 and 8 with a partner. Then share the responses as a class.

Share Phase, Question 6

- How is this question the same as Questions 3 through 5?
- How is this question different from Questions 3 through 5?
- What makes these questions more difficult than the previous questions?

Share Phase, Questions 7 and 8

Compare your answers from Question 7, parts (a) and (b) with the answers to Question 1, part (d) and Question 2, part (d). What do you notice?

6. Let \( y \) represent the number of DVDs Jeannie has. Write an algebraic expression that represents the number of DVDs for each friend.

a. John
   
   John has \( \frac{y}{3} \) DVDs.

b. Jenny
   
   Jenny has \( \frac{y}{3} - 4 \) DVDs.

c. Jack
   
   Jack has \( \frac{(y - 4)}{2} = \frac{y}{3} - \frac{4}{2} = \frac{y}{6} - 2 \) DVDs.

d. All four friends together
   
   They have \( y + \frac{y}{3} + \frac{y}{3} - 4 + \frac{y}{6} - 2 \) DVDs all together.

e. Simplify the expression you wrote in part (d).
   
   \( \frac{11}{6} y - 6 \)

7. Use your expression from part (e) to determine how many DVDs they have altogether if Jeannie has:

a. 72 DVDs.
   
   \( \frac{11}{6} (72) - 6 = 126 \)
   
   They have 126 DVDs.

b. 24 DVDs.
   
   \( \frac{11}{6} (24) - 6 = 38 \)
   
   They have 38 DVDs.

c. 36 DVDs.
   
   \( \frac{11}{6} (36) - 6 = 60 \)
   
   They have 60 DVDs.

d. 660 DVDs.
   
   \( \frac{11}{6} (660) - 6 = 1204 \)
   
   They have 1204 DVDs.

8. Write an algebraic expression to represent the number of DVDs for:

a. the males.
   
   The males have \( \frac{1}{2} y - 2 \).

b. the females.
   
   The females have \( \frac{4}{3} y - 4 \).
Grouping
Have students complete Questions 9 through 11 with a partner. Then share the responses as a class.

Share Phase, Questions 9 through 11
When did you have to use backward thinking to solve for the number of DVDs a friend had? Explain how your answer relates to the initial statements made at the beginning of the problem.

9. Let \( z \) represent the number of DVDs Jenny has. Write an algebraic expression that represents the number of DVDs for each friend.
   a. Jack
   \[
   \text{Jack has } \frac{z}{2} \text{ DVDs.}
   \]
   b. John
   \[
   \text{John has } z + 4 \text{ DVDs.}
   \]
   c. Jeannie
   \[
   \text{Jeannie has } 3(z + 4) \text{ or } 3z + 12 \text{ DVDs.}
   \]
   d. all four friends together
   \[
   \text{They have } z + \frac{z}{2} + z + 4 + 3(z + 4) \text{ DVDs.}
   \]
   e. Simplify the expression you wrote in part (d).
   \[
   \frac{11}{2}z + 16
   \]

10. Use your expression from part (e) to determine the number of DVDs they have altogether if Jenny has:
   a. 20 DVDs.
   \[
   \frac{11}{2}(20) + 16 = 126
   \]
   They have 126 DVDs.
   b. 24 DVDs.
   \[
   \frac{11}{2}(24) + 16 = 148
   \]
   They have 148 DVDs.
   c. 50 DVDs.
   \[
   \frac{11}{2}(50) + 16 = 291
   \]
   They have 291 DVDs.
   d. 34 DVDs.
   \[
   \frac{11}{2}(34) + 16 = 203
   \]
   They have 203 DVDs.

11. Write an algebraic expression to represent the number of DVDs for:
   a. the males.
   \[
   \text{The males have } \frac{3}{2}z + 4.
   \]
   b. the females.
   \[
   \text{The females have } 4z + 12.
   \]
Grouping
Have students complete Questions 12 through 14 with a partner. Then share the responses as a class.

Share Phase, Questions 12 through 14
What do the expressions in Question 3, part (e), Question 6, part (e), Question 9, part (e) and Question 12, part (e) have in common? How are they different?

12. Let $j$ represent the number of DVDs John has. Write an algebraic expression that represents the number of DVDs for each friend.
   a. Jenny
      Jenny has $j - 4$ DVDs.
   b. Jack
      Jack has $j - 4/2$ DVDs.
   c. Jeannie
      Jeanie has $3j$ DVDs.
   d. all four friends together
      They have $j - 4 + j + j - 4/2 + 3j$ DVDs.
   e. Simplify the expression you wrote in part (d).
      \[
      \frac{11}{2}j - 6
      \]

13. Use your expression from part (e) to determine the number of DVDs they have altogether if John has:
   a. 24 DVDs.
      \[
      \frac{11}{2}(24) - 6 = 126
      \]
      They have 126 DVDs.
   b. 8 DVDs.
      \[
      \frac{11}{2}(8) - 6 = 38
      \]
      They have 38 DVDs.
   c. 20 DVDs.
      \[
      \frac{11}{2}(20) - 6 = 104
      \]
      They have 104 DVDs.
   d. 60 DVDs.
      \[
      \frac{11}{2}(60) - 6 = 324
      \]
      They have 324 DVDs.

14. Write an algebraic expression to represent the number of DVDs for:
   a. the males.
      The males have $3j/2 - 2$.
   b. the females.
      The females have $4j - 4$. 
Problem 2
A context is provided that relates the number of MP3 songs different students own based upon the number of MP3 songs other students in the group own. The second problem is set up the same as the first problem, the only differences being the context, the number of relationships described in the problem, a minor change among the way the relationships are described, and the number of algebraic expressions students are asked to write.

Grouping
Have students complete Questions 1 through 6 with a partner. Then share the responses as a class.

Share Phase, Questions 1 through 6
- How are the given statements different from the previous problem’s given statements?
- Was it any easier beginning with the number of MP3 songs Melvin had or Marilyn had? Explain your reasoning.
- How could you check that your algebraic expressions are correct for Question 1, parts (a), (b), (c), and (d)?
- Explain the steps you used to simplify the expression in Question 1, part (e).
- Did anyone simplify the algebraic expression another way? If so, explain your steps.

Problem 2 Songs on an MP3 Player
Five friends have their own MP3 players.
- Marvin has 5 more songs on his MP3 than Melvin has on his.
- Marilyn has half as many songs on her MP3 as Marvin has on his.
- Maryanne has 3 more than twice the number of songs on her MP3 as Melvin has on his.
- Marty has 3 times as many songs on his MP3 as Marilyn has on hers.

1. Let \( x \) represent the number of songs on Melvin’s MP3 player. Write an algebraic expression that represents the number of songs on each friend’s MP3 player.
   a. Marvin
   \[ x + 5 \]
   b. Marilyn
   \[ \frac{x + 5}{2} \]
   c. Maryanne
   \[ 3 + 2x \]
   d. Marty
   \[ 3 \left( \frac{x + 5}{2} \right) \]
   e. all five friends together
   They have \( x + x + 5 + \frac{x + 5}{2} + 3 + 2x + 3 \left( \frac{x + 5}{2} \right) \)
   f. Simplify the expression you wrote in part (e).
   \[ 6x + 18 \]

2. Use your expression from part (f) to calculate the number of songs they have altogether if Melvin has:
   a. 15 songs.
   \[ 6(15) + 18 = 108 \]
   They have 108 songs.
   b. 47 songs.
   \[ 6(47) + 18 = 300 \]
   They have 300 songs.

3. Write an algebraic expression to represent the number of songs for:
   a. the males.
   \[ \frac{7}{2} x + \frac{25}{2} \]
   The males have \( \frac{7}{2} x + \frac{25}{2} \).
   b. the females.
   \[ \frac{5}{2} x + \frac{11}{2} \]
   The females have \( \frac{5}{2} x + \frac{11}{2} \).
4. Let $y$ represent the number of songs on Marilyn’s MP3 player. Write an algebraic expression that represents the number of songs on each friend’s MP3 player.

   a. Marvin
      \[2y\]

   b. Melvin
      \[2y - 5\]

   c. Maryanne
      \[3 + 2(2y - 5)\]

   d. Marty
      \[3y\]

   e. all five friends together
      They have $y + 2y + 2y - 5 + 3 + 2(2y - 5) + 3y$.

   f. Simplify the expression you wrote in part (e).
      \[12y - 12\]

5. Use your expression from part (f) to calculate the number of songs they have altogether if Marilyn has:

   a. 15 songs.
      \[12(15) - 12 = 168\]
      They have 168 songs.

   b. 20 songs.
      \[12(20) - 12 = 228\]
      They have 228 songs.

6. Write an algebraic expression to represent the number of songs for:

   a. the males.
      The males have $7y - 5$.

   b. the females.
      The females have $5y - 7$.

Be prepared to share your solutions and methods.
Follow Up

Assignment
Use the Assignment for Lesson 8.6 in the Student Assignments book. See the Teacher’s Resources and Assessments book for answers.

Skills Practice
Refer to the Skills Practice worksheet for Lesson 8.6 in the Student Assignments book for additional resources. See the Teacher’s Resources and Assessments book for answers.

Assessment
See the Assessments provided in the Teacher’s Resources and Assessments book for Chapter 8.

Check for Students’ Understanding
1. Yolanda is 4 years older than Xenia.
   Zack is $\frac{1}{2}$ of Yolanda’s age.
   a. Let $x$ represent Xenia’s age. Write expressions that represent the ages of Yolanda and Zack.
      Yolanda’s age $= x + 4$
      Zack’s age $= \frac{1}{2}(x + 4)$
   
   b. Let $y$ represent Yolanda’s age. Write expressions that represent the ages of Xenia and Zack.
      Xenia’s age $= y - 4$
      Zack’s age $= \frac{1}{2}y$

   c. Let $z$ represent Zack’s age. Write expressions that represent the ages of Yolanda and Xenia.
      Yolanda’s age $= 2z$
      Xenia’s age $= 2z - 4$
2. Michael has 5 times as much money as Nancy. Kathy has 18 more dollars than Michael.

   a. Let $n$ represent the amount of money Nancy has. Write expressions that represent the amount of money Michael has and the amount of money that Kathy has.

   - Michael’s money = $5n$
   - Kathy’s money = $5n + 18$

   b. Let $m$ represent the amount of money Michael has. Write expressions that represent the amount of money Nancy has and the amount of money that Kathy has.

   - Nancy’s money = $\frac{1}{5}m$
   - Kathy’s money = $m + 18$

   c. Let $k$ represent the amount of money Kathy has. Write expressions that represent the amount of money Michael has and the amount of money that Nancy has.

   - Michael’s money = $k - 18$
   - Nancy’s money = $\frac{1}{5}(k - 18)$
Chapter 8  Summary

Key Terms
- sequence (8.1)
- term (8.1)
- simplify (8.2)
- like terms (8.2)
- equivalent expressions (8.4)

Properties
- Commutative Property of Addition (8.2)
- Commutative Property of Multiplication (8.2)
- Associative Property of Addition (8.2)
- Associative Property of Multiplication (8.2)
- Distributive Property of Multiplication over Addition (8.3)
- Distributive Property of Multiplication over Subtraction (8.3)
- Distributive Property of Division over Addition (8.3)
- Distributive Property of Division over Subtraction (8.3)

8.1  Predicting the Next Term in a Sequence

A sequence is a pattern involving an ordered arrangement of numbers, geometric figures, letters, or other objects. A term is a number, variable, or product of numbers and variables. The first term is the first object or number in the sequence, the second term is the second object or number in the sequence, and so on. Look at the relationship between adjacent known terms and apply that same relationship to determine the next terms.

Example

Write the missing terms in the sequence.

Number of ants: 1, 2, 3, 4, 5, 6

Number of legs: 6, 12, 18, ____, ____ , ____

Each ant has 6 legs. So, each time an ant is added, add 6 to the number of legs.

The missing terms of the sequence are: 24, 30, 36.
Writing an Algebraic Expression to Represent a Sequence or Situation

Whenever you perform the same mathematical process over and over, you can write a mathematical phrase, called an algebraic expression, to represent the situation. An algebraic expression is a mathematical phrase involving at least one variable and sometimes numbers and operation symbols.

Examples

The algebraic expression $100 + 3n$, where $n$ is the term number, represents the sequence shown.
103, 106, 109, 112,...

The algebraic expression $2.49p$ represents the cost of $p$ pounds of apples.
One pound of apples costs $2.49.

Using the Associative and Commutative Properties of Addition and Multiplication to Simplify Expressions

The Commutative Properties of Addition and Multiplication state that the order in which you add or multiply two or more numbers does not affect the sum or the product. The Associative Properties of Addition and Multiplication state that changing the grouping of the terms in an addition or multiplication problem does not change the sum or the product. Use these rules to group addends or factors into expressions that are easy to compute in your head.

Example

$7 + 9 + 23 + 11 = (7 + 23) + (9 + 11)$

\[= 30 + 20\]

\[= 50\]
8.2 Using Algebra Tiles to Simplify Algebraic Expressions

To simplify an expression is to use the rules of arithmetic and algebra to rewrite the expression as simply as possible. An expression is in simplest form if all like terms have been combined. Like terms in an algebraic expression are two or more terms that have the same variable raised to the same power. Only the numerical coefficients of like terms are different. In an algebraic expression, a numerical coefficient is the number multiplied by a variable.

Example

You can use algebra tiles to represent each term in the algebraic expression. Combine like tiles and describe the result.

\( (3x + 2) + (5y + 3) + (x + 2y) \)

\[
\begin{align*}
\text{x} & \quad \text{x} & \quad 1 \\
\text{y} & \quad \text{y} & \quad 1 \\
\text{y} & \quad \text{y} & \quad 1 \\
\text{x} & \quad \text{x} & \quad 1 \\
\end{align*}
\]

\[4x + 7y + 5\]

8.3 Modeling the Distributive Property Using Algebra Tiles

The Distributive Property of Multiplication can be modeled by placing the tiles that match the first factor along the left side of a grid and the tiles that match the second factor along the top of a grid. The Distributive Property of Division can be modeled by separating a model of an expression into parts with an equal number of each type of tile.

Examples

a. \(3(2x + 1) = 6x + 3\)

\[
\begin{array}{c|c|c|c|c|c|c|c|c|c|c}
\text{x} & \text{x} & 1 \\
1 & \text{x} & \text{x} & 1 \\
1 & \text{x} & \text{x} & 1 \\
1 & \text{x} & \text{x} & 1 \\
\end{array}
\]

b. \(6y + 12 = 3(2y + 4)\)

\[
\begin{array}{c|c|c|c|c|c|c|c|c|c|c}
\text{y} & 1 & 1 \\
\text{y} & 1 & 1 \\
\text{y} & 1 & 1 \\
\text{y} & 1 & 1 \\
\text{y} & 1 & 1 \\
\text{y} & 1 & 1 \\
\end{array}
\]
Simplifying Algebraic Expressions Using the Distributive Property of Multiplication

The Distributive Property of Multiplication over Addition states that for any numbers $a$, $b$, and $c$, $a \times (b + c) = a \times b + a \times c$. The Distributive Property of Multiplication over Subtraction states that if $a$, $b$, and $c$ are real numbers, then $a \times (b - c) = a \times b - a \times c$.

Examples

\[
9(4x + 2) \quad \quad \quad \quad \quad 4(2x - 6) \\
36x + 18 \quad \quad \quad \quad \quad 8x - 24
\]

Simplifying Algebraic Expressions Using the Distributive Property of Division

The Distributive Property of Division over Addition states that if $a$, $b$, and $c$ are real numbers and $c \neq 0$, then $\frac{a + b}{c} = \frac{a}{c} + \frac{b}{c}$. The Distributive Property of Division over Subtraction states that if $a$, $b$, and $c$ are real numbers and $c \neq 0$, then $\frac{a - b}{c} = \frac{a}{c} - \frac{b}{c}$.

Examples

\[
\frac{2x + 8}{2} = \frac{2x}{2} + \frac{8}{2} \quad \quad \quad \quad \quad \frac{3x - 7}{3} = \frac{3x}{3} - \frac{7}{3} \\
= x + 4 \quad \quad \quad \quad \quad \quad = x - \frac{7}{3}
\]
8.4 Determining If Two Expressions Are Equivalent Using a Table or a Graph

You can determine if two expressions are equivalent in one of two ways: (1) by calculating values for each or (2) by graphing each. If the output values of each expression are the same for each input, then the expressions are equivalent. If the graph of each expression is the same, then the expressions are equivalent.

Example

The table shows that the expression \((x + 12) + (4x - 9)\) and the expression \(5x + 3\) are equivalent.

<table>
<thead>
<tr>
<th>(x)</th>
<th>((x + 12) + (4x - 9))</th>
<th>(5x + 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>13</td>
<td>13</td>
</tr>
</tbody>
</table>

The outputs for each expression are the same, so \((x + 12) + (4x - 9) = 5x + 3\).

The graph shows that the expression \(3(x + 3) - x\) and the expression \(3x + 3\) are not equivalent.

The graphs shown are not the same line, so \(3(x + 3) - x \neq 3x + 3\).
8.4 Writing and Solving an Expression Using the Distributive Property

You can use the Distributive Property to solve a problem for any input.

Example

Two friends split the cost of a rental car for three days. Each day the car costs $26 for insurance and fees plus 18¢ per mile driven. If the two friends plan to drive the same number of miles each day, how much will each one pay?

Let \( m \) represent the number of miles driven each day.

\[
\frac{3(26 + 0.18m)}{2} = \frac{78 + 0.54m}{2} = \frac{78}{2} + \frac{0.54m}{2} = 39 + 0.27m
\]

If each friend drives 250 miles each day, how much will each friend pay?

\[39 + 0.27(250) = 106.5, \text{ so each friend will pay } \$106.50.\]

8.5 Combining Like Terms Using Algebra Tiles

Algebra tiles can be used to model algebraic expressions. The tiles can then be rearranged so that multiple terms with identical tiles, called like terms, can be combined into one term. After all of the like terms are combined, the original algebraic expression can be rewritten with the fewest possible terms.

Example

Represent the algebraic expression \(3x + 5 + x - 1\) using algebra tiles and the operation symbols. Sketch your model.
Rearrange the tiles to combine like terms and to reduce the number of terms to the fewest possible terms.

\[ x + 1 + x + 1 + x + 1 + x + 1 \]

Write the algebraic expression represented by the model.

\[ 4x + 4 \]

### 8.6 Using Algebraic Expressions to Analyze and Solve Problems

Many real-life situations can be represented using algebraic expressions. The algebraic expressions can then be used to answer questions about the situation.

**Example**

Sophia, Hector, Gavin and Jenna are comparing their video game collection. Sophia has 5 more games than Hector. Gavin has twice as many games as Sophia. Jenna has 12 fewer games than Gavin.

**a.** Let \( x \) represent the number of video games that Hector has. Write an algebraic expression that represents the number of video games that each person has.

- Hector: \( x \)
- Sophia: \( x + 5 \)
- Gavin: \( 2(x + 5) \)
- Jenna: \( 2(x + 5) - 12 \)

All 4 friends together:

\[ x + x + 5 + 2(x + 5) + 2(x + 5) - 12 \]

\[ = 6x + 13 \]
b. Use your expression to determine the number of video games they have altogether if Hector has:

18 video games

\[6(18) + 13\]

\[= 121\]

They have 121 video games.

46 video games

\[6(46) + 13\]

\[= 289\]

They have 289 video games.