This photo shows a classic optical illusion called the Necker Cube. It’s an example of an impossible object. Optical illusions are often helpful to scientists who study how we see the world around us. Can you see why this cube is “impossible”?

3 TRANSLATIONS, REFLECTIONS, AND ROTATIONS

3.1 SLIDING RIGHT, LEFT, UP, DOWN, AND DIAGONALLY
Translations Using Geometric Figures ......................... 117

3.2 ROUND AND ROUND WE GO!
Rotations of Geometric Figures on the Coordinate Plane ...........................................127

3.3 MIRROR, MIRROR
Reflections of Geometric Figures on the Coordinate Plane ...........................................135

3.4 SLIDE, FLIP, TURN!
Translations, Rotations, and Reflections of Triangles ...................................................143

3.5 ALL THE SAME TO YOU
Congruent Triangles .....................................................155
### Chapter 3 Overview

The chapter focuses on translations, rotations, and reflections of geometric figures on a coordinate plane.

<table>
<thead>
<tr>
<th>Lesson</th>
<th>TEKS</th>
<th>Pacing</th>
<th>Highlights</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Translations Using Geometric Figures</td>
<td>8.10.A 8.10.B 8.10.C</td>
<td>1</td>
</tr>
<tr>
<td>3.2</td>
<td>Rotations of Geometric Figures on the Coordinate Plane</td>
<td>8.10.A 8.10.C</td>
<td>1</td>
</tr>
<tr>
<td>3.3</td>
<td>Reflections of Geometric Figures on the Coordinate Plane</td>
<td>8.10.A 8.10.C</td>
<td>1</td>
</tr>
<tr>
<td>3.4</td>
<td>Translations, Rotations, and Reflections of Triangles</td>
<td>8.10.B 8.10.C</td>
<td>1</td>
</tr>
<tr>
<td>3.5</td>
<td>Congruent Triangles</td>
<td>8.10.A 8.10.B</td>
<td>1</td>
</tr>
</tbody>
</table>
## Skills Practice Correlation for Chapter 3

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Problem Set</th>
<th>Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>3.1 Translations Using Geometric Figures</strong></td>
<td>Vocabulary</td>
<td>1 - 10 Determine coordinates of images following a given translation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11 - 16 Translate figures on a coordinate plane</td>
</tr>
<tr>
<td><strong>3.2 Rotations of Geometric Figures on the Coordinate Plane</strong></td>
<td>Vocabulary</td>
<td>1 - 10 Rotate given figures on a coordinate plane</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11 - 16 Rotate figures given the angle of rotation</td>
</tr>
<tr>
<td><strong>3.3 Reflections of Geometric Figures on the Coordinate Plane</strong></td>
<td>Vocabulary</td>
<td>1 - 10 Determine vertices of reflected images</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11 - 20 Reflect figures over a specified axis or line</td>
</tr>
<tr>
<td><strong>3.4 Translations, Rotations, and Reflections of Triangles</strong></td>
<td>Vocabulary</td>
<td>1 - 10 Perform given transformations</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11 - 20 Determine coordinates of a triangle's image after given transformations</td>
</tr>
<tr>
<td></td>
<td></td>
<td>21 - 30 Describe transformations used to formed triangles</td>
</tr>
<tr>
<td><strong>3.5 Congruent Triangles</strong></td>
<td>Vocabulary</td>
<td>1 - 10 Identify transformations used to create given triangles and list corresponding sides and angles</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11 - 18 List corresponding sides and angles of triangles from given congruence statements</td>
</tr>
</tbody>
</table>
3.1

Sliding Right, Left, Up, Down, and Diagonally
Translations Using Geometric Figures

Learning Goals
In this lesson, you will:
- Translate geometric figures horizontally.
- Translate geometric figures vertically.
- Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of translations.

Essential Ideas
- A transformation is the mapping, or movement, of all the points of a figure in a plane according to a common operation.
- A translation is a transformation that slides each point of a figure that same distance and direction. The new figure created from the translation is called the image.

Texas Essential Knowledge and Skills for Mathematics
Grade 8
(10) Two-dimensional shapes. The student applies mathematical process standards to develop transformational geometry concepts. The student is expected to:
- (A) generalize the properties of orientation and congruence of rotations, reflections, translations, and dilations of two-dimensional shapes on a coordinate plane
- (B) differentiate between transformations that preserve congruence and those that do not
- (C) explain the effect of translations, reflections over the x- or y-axis, and rotations limited to 90°, 180°, 270°, and 360° as applied to two-dimensional shapes on a coordinate plane using an algebraic representation

Key Terms
- transformation
- translation
- image
- pre-image

Materials
Scissors
Straightedge
Overview
In the introduction to this chapter, students cut out a trapezoid, two triangles, and a parallelogram. They will use these figures to explore horizontal, vertical, and diagonal translations throughout the lessons.

The term transformation is defined, and students horizontally and vertically translate a parallelogram on a coordinate plane. The terms translation and pre-image are then introduced. In the next activity, students horizontally and vertically translate a triangle and identify the coordinates of the vertices of each new image resulting from the translations.

Next, they vertically and horizontally translate an isosceles triangle. Essentially, they have performed a diagonal translation. The process is repeated with a trapezoid. Students will identify the changes that take place to the x-value and the y-value of each ordered pair that represents a vertex as the figure moves through a translation.
1. Identify the ordered pairs associated with each of the five vertices of the star.

(0, 11), (9, 3), (5, −9), (−5, −9), (−9, 3)
In this lesson, you will:
- Translate geometric figures horizontally.
- Translate geometric figures vertically.
- Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of translations.

Key Terms
- transformation
- translation
- image
- pre-image

To begin this chapter, cut out the figures shown on this page. You will have a trapezoid, two triangles, and a parallelogram. You will be using these figures in several lessons. What do you know about these shapes?

Note
Students need to cut out the four figures in the introduction to the chapter. These models are used throughout the lessons of this chapter.

ELL Tip
Pre-teach the math terms that will be used in this chapter by helping students to create a graphic organizer for the different types of transformations. For example:

- Translation is a slide. Draw an example of an image moving in a straight line. Highlight the “sl” in both words to show a connection.
- Rotation is a turn. Draw an example of an image turning. Highlight the “t” in both words to show a connection.
- Reflection is a flip. Draw an example of a flipped image or an image in a mirror. Highlight the “f” in both words to show a connection.
Problem 1

The term transformation is defined and students use the cut out parallelogram for the first activity. Students slide the model of the parallelogram horizontally and vertically on a coordinate plane. They conclude that the parallelograms created are all congruent but in different locations on the plane. Next students will perform horizontal and vertical slides on the coordinate plane using both triangular models. They list the ordered pairs of the vertices of the pre-image and image noting the changes in the x-values and y-values resulting from the movements of the figure.

Materials
Scissors
Straightedge

Grouping
Have students complete Questions 1 and 2 with a partner. Then share the responses as a class.

Share Phase, Question 1

- Which quadrant of the coordinate plane is the location of the original figure?
- Which quadrant of the coordinate plane is the location of the first image or figure 1?
- Which quadrant of the coordinate plane is the location of the second image or figure 2?
- Which quadrant of the coordinate plane is the location of the third image or figure 3?

Problem 1 Sliding to the Right, Left, Up, and Down

Let’s explore different ways to move, or transform, figures across a coordinate plane.
A transformation is the mapping, or movement, of all the points of a figure in a plane according to a common operation.

1. Look at the parallelogram shown on the coordinate plane.

   a. Place your parallelogram on the original figure on the coordinate plane shown and slide it 5 units to the left. Trace your parallelogram on the coordinate plane, and label it Figure 1.
   b. Place your parallelogram on the original figure on the coordinate plane shown and slide it 5 units down. Trace your parallelogram on the coordinate plane, and label it Figure 2.
   c. Place your parallelogram on Figure 1 on the coordinate plane and slide it 5 units down. Trace your parallelogram on the coordinate plane, and label it Figure 3.
   d. Describe how all of the parallelograms you traced on the coordinate plane compare with each other.

   All the parallelograms I traced are congruent, but they are in different places on the coordinate plane.
Chapter 3  Translations, Reflections, and Rotations

Share Phase, Question 2

- Is a movement to the left on the coordinate plane considered a negative or positive directional movement?
- Is a movement down on the coordinate plane considered a negative or positive directional movement?
- How do you know all of the parallelograms traced are congruent to each other?

2. Recall that two geometric figures are considered congruent when they are the same size and the same shape.
   a. Did sliding the parallelogram either up or down on the coordinate plane change the size or shape of the parallelogram?
      No. Sliding the parallelogram preserves the size and shape of the geometric figure.
   b. Are Figure 1, Figure 2, and Figure 3 all congruent to the original parallelogram shown on the coordinate plane? Explain your reasoning.
      Yes. I know that Figures 1, 2, and 3 are all congruent to the original parallelogram shown on the coordinate plane because I traced a model of the parallelogram shown to create all three figures, so the size and shape of the parallelogram were not altered.

3. Look at the triangle shown on the coordinate plane.

   a. List the ordered pairs for the vertices of \( \triangle ABC \).
      The ordered pairs for the vertices of \( \triangle ABC \) are \( A(1, 2), B(2, 4), \) and \( C(4, 2) \).
Share Phase, Question 3, parts (b) through (i)

- Is the vertical translation of the triangle an upward or a downward movement?
- Is the horizontal translation of the triangle a left or a right movement?
- How did you know where to label the A’ in the first image?
- How did you know where to label the B’ in the first image?
- How did you know where to label the C’ in the first image?
- Could you have predicted the ordered pairs of the image without graphing it? How?
- How did you determine the ordered pairs for the vertices of triangle A’B’C’?
  - Which values of the ordered pairs did not change? Why?
- How did you determine the ordered pairs for the vertices of triangle A”B”C”?
  - Which values of the ordered pairs did not change? Why?
- When the figure was slid horizontally, why did the x-values change?
- When the figure was slid vertically, why did the y-values change?
- What combination of slides would result in a diagonal translation?
- Does a slide to the left followed by a slide down result in a diagonal translation?

b. Place your triangle on \( \triangle ABC \), and translate it \(-6\) units vertically. Trace the new triangle, and label the vertices \( A', B', \) and \( C' \) in \( \triangle A'B'C' \) so the vertices correspond to the vertices \( A, B, \) and \( C \) in \( \triangle ABC \).

c. List the ordered pairs for the vertices of \( \triangle A'B'C' \).
  The ordered pairs for the vertices of \( \triangle A'B'C' \) are \( A'(1, -4), B'(2, -2), \) and \( C'(4, -4) \).

d. Place your triangle on \( \triangle ABC \), and translate it \(-6\) units horizontally. Trace the new triangle, and label the vertices \( A', B', \) and \( C' \) in \( \triangle A'B'C' \) so the vertices correspond to the vertices \( A, B, \) and \( C \) in \( \triangle ABC \).

e. List the ordered pairs for the vertices of \( \triangle A'B'C' \).
  The ordered pairs for the vertices of \( \triangle A'B'C' \) are \( A'(-5, 2), B'(-4, 4), \) and \( C'(-2, 2) \).

f. Compare the ordered pairs in \( \triangle ABC \) and \( \triangle A'B'C' \). How are the values in the ordered pairs affected by the translation?
  The \( x \)-values did not change, but the \( y \)-values are six less than in the original ordered pairs, which is the same value as the translation.

g. Compare the ordered pairs in \( \triangle ABC \) and \( \triangle A'B'C' \). How are the values in the ordered pairs affected by the translation?
  The \( y \)-values did not change, but the \( x \)-values are six less than in the original ordered pairs, which is the same value as the translation.

h. If you were to translate \( \triangle ABC \) \(10\) units vertically to form \( \triangle DEF \), what would be the ordered pairs of the corresponding vertices?
  The ordered pairs of the corresponding vertices of \( \triangle DEF \) would be \( D(1, 12), E(2, 14), \) and \( F(4, 12) \).

i. If you were to translate \( \triangle ABC \) \(10\) units horizontally to form \( \triangle GHJ \), what would be the ordered pairs of the corresponding vertices?
  The ordered pairs of the corresponding vertices of \( \triangle GHJ \) would be \( G(11, 2), H(12, 4), \) and \( J(14, 2) \).
4. Recall that two geometric figures are considered congruent when they are the same size and the same shape.
   a. Did sliding the triangle either up or down on the coordinate plane change the size or shape of the triangle?
      No. Sliding the triangle preserves the size and shape.

   b. Are both of the triangles you drew congruent to the triangle shown on the coordinate plane? Explain your reasoning.
      I know that the triangles are congruent to the triangle shown on the coordinate plane because I traced a model of the triangle shown to create the other two triangles, so the size and shape of the triangle were not altered.

5. Look at the triangle shown on the coordinate plane.

   a. List the ordered pairs for the vertices of \( \triangle ABC \).
      \( A(1, 1), B(3, 4), C(5, 1) \)

   b. Place your triangle on \( \triangle ABC \), and translate it \( -5 \) units vertically. Trace the new triangle, and label the vertices \( A', B', \) and \( C' \) in \( \triangle A'B'C' \) so the vertices correspond to the vertices \( A, B, \) and \( C \) in \( \triangle ABC \).

   c. List the ordered pairs for the vertices of \( \triangle A'B'C' \).
      \( A'(1, -4), B'(3, -1), C'(5, -4) \)
Share Phase,
Questions 5, parts (d) through (i) and 6

- How did you determine the ordered pairs for the vertices of triangle A'B'C'?
- Which values of the ordered pairs did not change? Why?
- When the figure was slid horizontally, why did the x-values change?
- When the figure was slid vertically, why did the y-values change?
- What combination of slides would result in a diagonal translation?
- Does a slide to the left followed by a slide down result in a diagonal translation?

- Place your triangle on $\triangle ABC$, and translate it –5 units horizontally. Trace the new triangle, and label the vertices $A'$, $B'$, and $C'$ in $\triangle A'B'C'$ so the vertices correspond to the vertices $A$, $B$, and $C$ in $\triangle ABC$.

- List the ordered pairs for the vertices of $\triangle A'B'C'$.
  $A'(-4, 1)$, $B'(-2, 4)$, $C'(0, 1)$

- Compare the ordered pairs in $\triangle ABC$ and $\triangle A'B'C'$. How are the values in the ordered pairs affected by the translation?
  All of the y-values are 5 less. The x-values stayed the same.

- Compare the ordered pairs in $\triangle ABC$ and $\triangle A'B'C'$. How are the values in the ordered pairs affected by the translation?
  All of the x-values are 5 less. The y-values stayed the same.

- If you were to translate $\triangle ABC$ 10 units vertically to form $\triangle DEF$, what would be the ordered pairs of the corresponding vertices?
  $D(1, 11)$, $E(3, 14)$, $F(6, 11)$

- If you were to translate $\triangle ABC$ 10 units horizontally to form $\triangle GHJ$, what would be the ordered pairs of the corresponding vertices?
  $G(11, 1)$, $H(13, 4)$, $J(15, 1)$

6. Are both triangles congruent to the original triangle shown on the coordinate plane? Explain your reasoning.

   Yes. I know that both triangles are congruent to the original triangle shown on the coordinate plane because I traced a model of the triangle shown to create the two figures, so the size and shape of the triangle were not altered.
Problem 2
Students slide the model of a trapezoid horizontally and vertically on a coordinate plane, and list the ordered pairs of the images. Noting the change in the x-value and the y-values resulting from the translation, they are able to list ordered pairs that result from a different slide without physically performing the translation. Students conclude that the pre-images and image of the trapezoid are all congruent.

Grouping
Have students complete Questions 1 and 2 with a partner. Then share the responses as a class.

Share Phase, Question 1, parts (a) through (e)
- How did you determine the ordered pairs for the vertices of trapezoid A'B'C'D'?
- Which values of the ordered pairs did not change? Why?
- How did you determine the ordered pairs for the vertices of triangle A''B''C''D''?
- Which values of the ordered pairs did not change? Why?

Can you predict what will happen to the ordered pairs of the trapezoid?

Problem 2  Translating a Trapezoid

1. Look at the trapezoid shown on the coordinate plane.

a. List the ordered pairs for the vertices of trapezoid ABCD.
   A(0, 1), B(1, 2.5), C(3, 2.5), D(5, 1)

b. Place your trapezoid on trapezoid ABCD, and translate it –5 units vertically. Trace the new trapezoid, and label the vertices A', B', C', and D' in trapezoid A'B'C'D' so the vertices correspond to the vertices A, B, C, and D in trapezoid ABCD.

c. List the ordered pairs for the vertices of trapezoid A'B'C'D':
   A'(0, -4), B'(1, -2.5), C'(3, -2.5), D'(5, -4)

d. Place your trapezoid on trapezoid ABCD, and translate it –5 units horizontally. Trace the new trapezoid, and label the vertices A'', B'', C'', and D'' in trapezoid A''B''C''D'' so the vertices correspond to the vertices A, B, C, and D in trapezoid ABCD.

e. List the ordered pairs for the vertices of trapezoid A''B''C''D'':
   A''(−5, 1), B''(−4, 2.5), C''(−2, 2.5), D''(0, 1)
Share Phase, Question 1, parts (f) through (i)

- Did you have to physically slide the trapezoid to determine the ordered pairs when the trapezoid is translated 10 units vertically? Why or why not?
- Did you have to physically slide the trapezoid to determine the ordered pairs when the trapezoid is translated 10 units horizontally? Why or why not?

f. Compare the ordered pairs in trapezoid ABCD and trapezoid A'B'C'D'. How are the values in the ordered pairs affected by the translation?
   All of the y-values are 5 less. The x-values stayed the same.

g. Compare the ordered pairs in trapezoid ABCD and trapezoid A'B'C'D'. How are the values in the ordered pairs affected by the translation?
   All of the x-values are 5 less. The y-values stayed the same.

h. If you were to translate trapezoid ABCD 10 units vertically to form trapezoid DEFG, what would be the ordered pairs of the corresponding vertices?
   D(0, 11), E(1, 12.5), F(3, 12.5), G(5, 11)

i. If you were to translate trapezoid ABCD 10 units horizontally to form trapezoid HJKM, what would be the ordered pairs of the corresponding vertices?
   H(10, 1), J(11, 2.5), K(13, 2.5), M(15, 1)

2. Recall that two geometric figures are considered congruent when they are the same size and the same shape.

a. Did sliding the trapezoid either up or down on the coordinate plane change the size or shape of the trapezoid?
   No. Sliding the trapezoid preserves the size and shape of the geometric figure.

b. Are both trapezoids congruent to the original trapezoid shown on the coordinate plane? Explain your reasoning.
   Yes. I know that both trapezoids are congruent to the original trapezoid shown on the coordinate plane because I traced a model of the trapezoid shown to create the two figures, so the size and shape of the trapezoid were not altered.
Talk the Talk
Students explain why the pre-image and the image that results from a translation are congruent figures.

Grouping
Have students complete Questions 1 and 2 with a partner. Then share the responses as a class.

1. Are all images, or new figures that result from a translation, always congruent to the original figure? Explain your reasoning.
   Yes, all images resulting from a translation are congruent to the original figure, because by definition, a translation is a transformation that slides each corresponding point of the original figure the same distance and in the same direction. This does not change the size or shape of the original geometric figure.

2. For any real number \( c \) or \( d \), describe how the ordered pair \((x, y)\) of any original figure will change when translated:
   a. horizontally \( c \) units. How do you know if the image translated to the left or to the right?
      \((x + c, y)\)
      If \( c < 0 \), then the figure will translate to the left.
      If \( c > 0 \), then the figure will translate to the right.
   b. vertically \( d \) units. How do you know if the image translated up or down?
      \((x, y + d)\)
      If \( d < 0 \), then the figure will translate down.
      If \( d > 0 \), then the figure will translate up.

Be prepared to share your solutions and methods.
Follow Up

Assignment
Use the Assignment for Lesson 3.1 in the Student Assignments book. See the Teacher's Resources and Assessments book for answers.

Skills Practice
Refer to the Skills Practice worksheet for Lesson 3.1 in the Student Assignments book for additional resources. See the Teacher's Resources and Assessments book for answers.

Assessment
See the Assessments provided in the Teacher's Resources and Assessments book for Chapter 3.

Check for Students' Understanding

1. Graph the given coordinates to transfer the star onto the coordinate plane.
   \((0, 11), (2, 3), (9, 3), (4, -1), (5, -9), (0, -3), (-5, -9), (-4, -1), (-9, 3), (-2, 3)\)

2. Connect the coordinates to form the star and also sketch the circle and the heart on the star.

3. Without graphing, if the star was horizontally translated five units to the right, what are the new coordinates of the vertex of the star closest to the dot?
   \((14, 3)\)
4. Without graphing, if the star was vertically translated six units to the down, what are the new coordinates of the vertex of the star closest to the heart?
   
   \((5, -15)\)

5. Without graphing, if the star was horizontally translated seven units to the left and vertically translated two units to the up, what are the new coordinates of the vertex of the star closest to the dot?
   
   \((2, 5)\)
3.2 Rounding and Round We Go!
Rotations of Geometric Figures on the Coordinate Plane

Learning Goal
In this lesson, you will:
- Rotate geometric figures on the coordinate plane.

Key Terms
- rotation
- angle of rotation
- point of rotation

Essential Ideas
- A rotation is a transformation that turns a figure clockwise or counterclockwise about a fixed point for a given angle, and a given direction.
- An angle of rotation is the amount of clockwise or counterclockwise rotation about a mixed point.
- The point of rotation can be a point on the figure, in the figure, or not on the figure.

Texas Essential Knowledge and Skills for Mathematics
Grade 8
(10) Two-dimensional shapes. The student applies mathematical process standards to develop transformational geometry concepts. The student is expected to:

(A) generalize the properties of orientation and congruence of rotations, reflections, translations, and dilations of two-dimensional shapes on a coordinate plane

(C) explain the effect of translations, reflections over the x- or y-axis, and rotations limited to 90°, 180°, 270°, and 360° as applied to two-dimensional shapes on a coordinate plane using an algebraic representation

Materials
- Straightedge
- Protractor
- Pin
Overview
The terms rotation, angle of rotation, and point of rotation are introduced. Students rotate triangles, parallelograms, and trapezoids about points of rotation that are on the figure, in the figure, and not on the figure. Using the models created in the first lesson and placing a pin through the point of rotation, students will rotate figures.
Warm Up

1. Redraw each given figure:
   a. such that it is turned 180° clockwise.
      Before: After:

   b. such that it is turned 90° counterclockwise.
      Before: After:

   c. such that it is turned 90° clockwise.
      Before: After:

2. Describe how many degrees and in which direction (clockwise or counterclockwise) each figure was turned.
   a. Before: After:
      The figure was turned 90° clockwise or 180° counterclockwise.

   b. Before: After:
      The figure was turned 90° counterclockwise or 180° clockwise.

   c. Before: After:
      The figure was turned 180° clockwise or 180° counterclockwise.
Centrifuges are devices that spin material around a center point. Centrifuges are used in biology and chemistry, often to separate materials in a gas or liquid. Tubes are inserted into the device and, as it spins, heavier material is pushed to the bottom of the tubes while lighter material tends to rise to the top.

Human centrifuges are used to test pilots and astronauts. Can you think of other devices that work like centrifuges?
Problem 1
The pre-image of a triangle and the image that results from a rotation are given. Students use their triangle model to describe how the pre-image was transformed. The terms rotation, angle of rotation, and point of rotation are defined. Students are given graphs containing a triangle rotated about a vertex, rotated about a point not on the triangle, and rotated about a point in the triangle. Using the model of the triangle and a pin, they will perform similar rotations, and sketch the transformed image.

Materials
Straightedge
Protractor
Pin

Grouping
Have students complete Questions 1 through 4 with a partner. Then share the responses as a class.

Share Phase,
Questions 1 through 4

- Did you rotate ∆ABC to the right or to the left to align the model with the image?
- Does the rotation of the model to the right result in the same image when compared to rotating the model to the left?
- About how many degrees do you think the model was rotated to the right to produce the image?
- About how many degrees do you think the model was rotated to the left to produce the image?

Problem 1 What Is a Rotation?

You have considered what happens to shapes when you slide them up, down, left, or right. Let's explore what happens when you rotate a geometric figure.

1. Look at the triangles shown in the coordinate plane.

2. Place your triangle on ∆ABC. Without moving vertex A, “transform” the triangle into ∆AB'C'.

3. Describe how you transformed the triangle.
   I rotated (turned) it to the right or left until it was on top of the other.

4. Katie says that she can use translations to move triangle ABC to triangle AB'C'. Is she correct? Explain your reasoning.
   No. There is no way to move triangle ABC to triangle AB'C' using only sliding, or translations.

- Grab your shapes from the first lesson.

- Notice the triangles share a vertex.

128 • Chapter 3 Translations, Reflections, and Rotations
A rotation is a transformation that turns a figure about a fixed point for a given angle, called the angle of rotation, and a given direction. The angle of rotation is the amount of rotation about a fixed point, or point of rotation. Rotation can be clockwise or counterclockwise.

The point of rotation can be a point on the figure.

Or, it can be a point not on the figure.

It can also be a point in the figure.
Grouping
Have students complete Questions 5 through 8 with a partner. Then share the responses as a class.

Share Phase, Questions 5 through 8
• When using a pin to rotate a figure, what does the placement of the pin represent?
• When rotating your triangle model to produce ΔA'B'C', did you rotate the model clockwise or counterclockwise?
• When rotating your triangle model clockwise to produce ΔA'B'C', where did you place the pin with respect to side AC?
• Did everyone place the pin at the same location? Explain.
• When rotating your triangle model to produce ΔA'B'C'', did you rotate the model clockwise or counterclockwise?
• When rotating your triangle model clockwise to produce ΔA'B'C'', where did you place the pin with respect to side AC?
• Did everyone place the pin at the same location? Explain.
• If the pin were placed at different locations on side AC, how would this affect the outcome?

5. Use your triangle to rotate ΔABC in Question 1 by placing your triangle on the figure, putting a pin in it at vertex C, and then rotating your triangle first to the left and then to the right.
Students' rotations will vary. Please note that students will not trace their rotations on the coordinate plane for this question.

6. Using AC as one side of the angle, measure and draw ∠ACA to be 120°. Then, rotate your triangle clockwise to produce ΔCA'B'. Label your rotation in the coordinate plane.

7. Use your triangle to rotate ΔABC by placing your triangle on the figure, putting a pin in it at any point on side AC, and then rotating your triangle first clockwise and then counterclockwise. Trace one rotation you performed on the coordinate plane as ΔA'B'C''.

You will need your protractor.
Place a point at your point of rotation.
Problem 2
Students use the model of the parallelogram in this problem. They rotate the parallelogram about a point inside the figure on the coordinate plane provided and note that the rotation preserves the size and shape of the triangle. Students will conclude the image of the triangle is congruent to the pre-image of the triangle.

Grouping
Have students complete Questions 1 and 2 with a partner. Then share the responses as a class.

Share Phase, Question 1
- Was Question 1 an example of rotating a figure about the vertex of a parallelogram, about a point on the parallelogram, or about a point not on the parallelogram?

Problem 2 Rotating a Parallelogram

1. Use your parallelogram to rotate parallelogram ABCD by placing your parallelogram on the figure, putting a pin in it at any point in the interior of the parallelogram, and then rotating your parallelogram first clockwise and then counterclockwise. Trace one rotation you performed on the coordinate plane. Place a point at your center of rotation.

- Where did you place the pin?
- Did everyone place the pin at the same location?

8. Recall that two geometric figures are considered congruent when they are the same size and the same shape.
   a. Did rotating the triangle on the coordinate plane in any of the previous questions change the size or shape of the triangle?
      No. Rotating the triangle preserves the size and shape of the geometric figure.
   b. Is the image of the triangle that resulted from the rotation congruent to the triangle shown on the coordinate plane? Explain your reasoning.
      Yes. I know that the image is congruent to the triangle shown on the coordinate plane because I traced a model of the triangle shown to create the image, so the size and shape of the triangle were not altered.
Share Phase, Question 2

- Did different placements of the pin affect the size of the image? Explain.
- Did different placements of the pin affect the shape of the image? Explain.

Problem 3

Students use the model of the trapezoid in this problem. They rotate the trapezoid about a point not on the figure on the coordinate plane provided and note that the rotation preserves the size and shape of the trapezoid. Students will conclude the image of the trapezoid is congruent to the pre-image of the trapezoid.

Grouping

Have students complete Questions 1 and 2 with a partner. Then share the responses as a class.

Share Phase, Questions 1 and 2

- Was Question 1 an example of rotating a figure about the vertex of a trapezoid, about a point on the trapezoid, about a point in the trapezoid, or about a point not on the trapezoid?

Problem 3 Rotating a Trapezoid

1. Use your trapezoid to rotate trapezoid ABCD around point P by placing your trapezoid on the figure. Fold a piece of tape in half and tape it to both sides of the trapezoid, making sure that the tape covers point P. Put a pin in at point P, and rotate your parallelogram first clockwise and then counterclockwise. Trace one rotation you performed on the coordinate plane.

- What is an example of a rotating figure in the classroom?
- What is an example of a rotating figure in your home?
- What is an example of a rotating figure outside?
- Why is rotating figures a topic worth studying?
2. Recall that two geometric figures are considered congruent when they are the same size and the same shape.

a. Did rotating the trapezoid on the coordinate plane change the size or shape of the trapezoid?
   No. Rotating the trapezoid preserves the size and shape of the geometric figure.

b. Is the image of the trapezoid congruent to the trapezoid shown on the coordinate plane? Explain your reasoning.
   Yes. I know that the image is congruent to the trapezoid shown on the coordinate plane because I traced a model of the trapezoid shown to create the image, so the size and shape of the trapezoid were not altered.

Talk the Talk

1. Are all images, or new figures that result from a rotation, always congruent to the original figure? Explain your reasoning.
   Yes. All images resulting from a rotation are congruent to the original figure, because by definition, a rotation is a transformation that turns a figure about a fixed point for a given angle in a given direction. This does not change the size or shape of the original geometric figure.

Grouping

Have students complete Questions 1 and 2 with a partner. Then share the responses as a class.
2. Describe the point of rotation in each.

a. The point of rotation is in the figure.

b. The point of rotation is on the figure.

So is the point of rotation in the figure, on the figure, or not on the figure?

c. The point of rotation is not on the figure.

Be prepared to share your solutions and methods.
Follow Up

Assignment
Use the Assignment for Lesson 3.2 in the Student Assignments book. See the Teacher’s Resources and Assessments book for answers.

Skills Practice
Refer to the Skills Practice worksheet for Lesson 3.2 in the Student Assignments book for additional resources. See the Teacher’s Resources and Assessments book for answers.

Assessment
See the Assessments provided in the Teacher’s Resources and Assessments book for Chapter 3.

Check for Students’ Understanding

1. Graph the given coordinates to transfer the star onto the x-y coordinate plane. 
   \((0, 11), (2, 3), (9, 3), (4, -1), (5, -9), (0, -3), (-5, -9), (-4, -1), (-9, 3), (-2, 3)\)

2. Connect the coordinates to form the star and also sketch the circle and the heart on the star.

3. Without graphing, if the star was rotated 90° clockwise about the origin, what are the new coordinates of the vertex of the star closest to the dot?
   \((3, 9)\)
4. Without graphing, if the star was rotated 90° counterclockwise about the origin, what are the new coordinates of the vertex of the star closest to the heart?

\((9, -5)\)

5. Without graphing, if the star was rotated 180° clockwise about the origin, what are the new coordinates of the vertex of the star closest to the dot?

\((-9, -3)\)

6. Without graphing, if the star was rotated 180° clockwise about the origin, what are the new coordinates of the vertex of the star closest to the heart?

\((-5, 9)\)
Essential Ideas

- A reflection is a transformation that flips a figure over a reflection line.
- A reflection line is a line that acts as a mirror or perpendicular bisector so that corresponding points are the same distance from the mirror.
- When a geometric figure is reflected over the $y$-axis to form its image, the $x$-values of the ordered pairs of the vertices of the pre-image become opposites and the $y$-values of the ordered pairs of the pre-image remain the same.
- When a geometric figure is reflected over the $x$-axis to form its image, the $y$-values of the ordered pairs of the vertices of the pre-image become opposites and the $x$-values of the ordered pairs of the pre-image remain the same.

Texas Essential Knowledge and Skills for Mathematics

Grade 8

(10) Two-dimensional shapes. The student applies mathematical process standards to develop transformational geometry concepts. The student is expected to:

(A) generalize the properties of orientation and congruence of rotations, reflections, translations, and dilations of two-dimensional shapes on a coordinate plane

(C) explain the effect of translations, reflections over the $x$- or $y$-axis, and rotations limited to $90^\circ$, $180^\circ$, $270^\circ$, and $360^\circ$ as applied to two-dimensional shapes on a coordinate plane using an algebraic representation

Materials

- Straightedge
- Mirror
Overview
The terms reflection, and reflection line are introduced. Students reflect triangles and parallelograms over the $x$-axis and $y$-axis. They conclude that when reflected over these axes, the ordered pairs that represent the vertices of the pre-image are directly related to the ordered pairs of its image. When the figure is reflected over the $y$-axis, the $x$-values of the ordered pairs of the vertices of the pre-image become the opposites and the $y$-values of the ordered pairs of the pre-image remain the same. When the figure is reflected over the $x$-axis, the $y$-values of the ordered pairs of the vertices of the pre-image become the opposites and the $x$-values of the ordered pairs of the pre-image remain the same.
Warm Up

Ellie was making a paper pine tree. She folded a piece of paper in half and made some cuts in the paper. It looks like this:

1. Graph the given coordinates to transfer this figure onto an x-y coordinate plane.
   \((0, 0), (1, 2), (1, 0), (3, 4), (3, 0), (6, 6), (6, -1), (8, -1), (8, 0)\)

2. Ellie finished cutting out the tree and unfolded it. Use your graph to draw the other half of the tree. List the coordinates used to draw the other half of the tree.
   \((0, 0), (1, 2), (1, 0), (3, 4), (3, 0), (6, 6), (6, 1), (8, 1), (8, 0)\)

3. Compare the coordinates in Question 1 to the coordinates in Question 2.
   The coordinates are the same except each \(y\)-coordinate in the cut out have the opposite or negated value of the first half of the tree.
The astronauts aboard the Apollo Moon missions in 1969 through the 1970s did more than just play golf and take pictures. They also set up equipment on the Moon to help scientists measure the distance from the Moon to the Earth.

This equipment contained sets of mirrors, called retroreflectors. Scientists on Earth can now shoot laser beams at these mirrors and calculate the distance to the Moon by observing how long it takes the laser beam to "bounce back."
Problem 1
A triangle is reflected over the $y$-axis and then two triangles are reflected over the $x$-axis. Students conclude that the reflected triangles drawn on the coordinate plane are congruent. Next, they reflect a parallelogram over the $y$-axis using the $y$-axis as a perpendicular bisector. When students list the ordered pairs of the vertices of the pre-image and its image, they will notice the $x$-values are opposites and the $y$-values remained the same. They then reflect a parallelogram over the $x$-axis by using the $x$-axis as a perpendicular bisector. After listing the ordered pairs of the vertices of the pre-image and its image, students will notice the $y$-values are opposites and the $x$-values remained the same.

Materials
Straightedge
Mirror

Grouping
Ask a student to read the introduction to Problem 1 aloud. Discuss the definitions and complete Question 1 as a class.

Discuss Phase, Question 1, parts (a) through (d)
• Why do you think these two triangles appear to be congruent?
• If you cut out these two triangles, would one triangle fit exactly on top of the second triangle? How would you position the triangles so they fit exactly on top of each other?
Discuss Phase, Question 1, part (e)
- Did you use a mirror to draw the reflection of each of the triangles over the x-axis?
- Did you use a mirror to check the reflection you drew of each of the triangles over the x-axis?

Grouping
Have students complete Question 2 with a partner. Then share the responses as a class.

Share Phase, Question 2
- How did you determine the coordinates of the ordered pair representing point A’? B’? C’? D’?
- How do you know the y-axis is the perpendicular bisector of the segments formed by connecting A to A’, B to B’, C to C’, and D to D’?
- Why are the x-values of the images vertices the opposite of the x-values of the pre-images, when the point is reflected over the y-axis?
- Why are the y-values of the images vertices the same as the y-values of the pre-image vertices, when the point is reflected over the y-axis?

2. Reflect parallelogram ABCD, using the y-axis as the reflection line, to form parallelogram A’B’C’D’.

a. Connect each vertex of the original parallelogram to the corresponding vertex of the image with line segments ___, ___, ___, and ___.

b. Describe the relationship between the y-axis and each of the segments you drew.
   The y-axis is the perpendicular bisector of each of the segments.
Grouping
Have students complete Questions 3 and 4 with a partner. Then share the responses as a class.

Share Phase, Question 3
• How did you determine the coordinates of the ordered pair representing point A’? B’? C’? D’?
• Is the y-axis the perpendicular bisector of the segments formed by connecting A to A’, B to B’, C to C’, and D to D’ or is the x-axis the perpendicular bisector?

3. Reflect parallelogram ABCD across the x-axis by using the x-axis as a perpendicular bisector.

You might want to create a table to organize your ordered pairs.

a. List the ordered pairs for the vertices of the original parallelogram and the reflected image.
   The ordered pairs for the vertices of parallelogram ABCD are A(−4, 1), B(−2, 2), C(−2, −3), and D(−4, −4).
   The ordered pairs for the vertices of parallelogram A'B'C'D' are A'(4, 1), B'(2, 2), C'(2, −3) and D'(4, −4).

b. What do you notice about the ordered pairs of the vertices of the original figure and its reflection across the y-axis? The x-values are opposites, and the y-values are the same.

c. List the ordered pairs for the vertices of parallelogram ABCD and parallelogram A'B'C'D'.
   The ordered pairs for the vertices of parallelogram ABCD are A(−4, 1), B(−2, 2), C(−2, −3), and D(−4, −4).
   The ordered pairs for the vertices of parallelogram A'B'C'D' are A'(4, 1), B'(2, 2), C'(2, −3) and D'(4, −4).

You might want to create a table to organize your ordered pairs.
Share Phase, Question 4

- Why are the y-values of the images vertices the opposite of the y-values of the pre-image vertices, when the point is reflected over the x-axis?
- Why are the x-values of the images vertices the same as the x-values of the pre-image vertices, when the point is reflected over the x-axis?

Problem 2

Students reflect a triangle over a vertical line \( x = -1 \) and over a horizontal line \( y = 2 \) on a coordinate plane.

Grouping

Have students complete Questions 1 through 3 with a partner. Then share the responses as a class.

Share Phase, Question 1

- How is reflecting the triangle over the line \( x = -1 \) different than reflecting the triangle over the y-axis?
- How does this difference affect the coordinates of the image’s vertices? Are the x-values of the ordered pairs of the image still opposites? Did the y-values of the ordered pairs still remain the same?

b. What do you notice about the ordered pairs of the vertices of the original figure and its reflection across the x-axis?

The y-values are opposites, and the x-values are the same.

4. A triangle has vertices at \( A(-4, 3), B(1, 5), C(2, -2) \).

a. If this triangle is reflected across the x-axis, what would the ordered pairs of the reflection’s vertices be?

The reflection’s vertices would be \( A'(-4, -3), B'(1, -5), \) and \( C'(2, 2) \).

b. If this triangle is reflected across the y-axis, what would the ordered pairs of the reflection’s vertices be?

The ordered pairs of the reflected triangle would be \( A'(4, 3), B'(-1, 5), \) and \( C'(-2, -2) \).

Problem 2 Reflections Across Horizontal and Vertical Lines

1. Reflect the triangle across the line \( x = -1 \).
Share Phase, Question 2

- How is reflecting the triangle over the line $y = 2$ different than reflecting the triangle over the $x$-axis?
- How does this difference affect the resulting image?
- How does this difference affect the coordinates of the images vertices? Are the $y$-values of the ordered pairs of the image still opposites? Did the $x$-values of the ordered pairs still remain the same?
- What is an example of reflecting a figure in your classroom?
- What is an example of reflecting a figure in your home?
- What is an example of reflecting a figure outside?
- Why is reflecting figures a topic worth studying?

2. Reflect the triangle across the line $y = 2$.

3. Recall that geometric figures are considered congruent when they are the same size and the same shape.
   a. Did reflecting the triangle on the coordinate plane change the size or shape of the figure?
      No. Reflecting the triangle preserves the size and shape of the geometric figure. It just changes the orientation of the figure.
   b. Is the image of the reflection of the triangle congruent to the original figure shown on the coordinate plane? Explain your reasoning.
      Yes. The image of the reflection is congruent to the geometric figure shown on the coordinate plane because it is the mirror image, and a mirror does not distort the size or shape of a figure, it only changes the orientation.
Talk the Talk
Students explain why the pre-image and the image that result from a reflection are congruent figures. They are then given the pre-image and the image of a figure in three situations and will describe the line of reflection in each instance.

Grouping
Have students complete Questions 1 and 2 with a partner. Then share the responses as a class.

1. Are all images, or new figures that result from a reflection, always congruent to the original figure? Explain.
   Yes. All images resulting from a reflection are congruent to the original figure, because by definition, a reflection is a transformation that flips a figure across a reflection line, which acts much the same as a mirror. This does not change the size or shape of the original geometric figure.

2. Describe the line of reflection in each.
   a. The line of reflection is $x = -2$.
   b. The line of reflection is $y = 2$.

Be prepared to share your solutions and methods.
Follow Up

Assignment
Use the Assignment for Lesson 3.3 in the Student Assignments book. See the Teacher’s Resources and Assessments book for answers.

Skills Practice
Refer to the Skills Practice worksheet for Lesson 3.3 in the Student Assignments book for additional resources. See the Teacher’s Resources and Assessments book for answers.

Assessment
See the Assessments provided in the Teacher’s Resources and Assessments book for Chapter 3.

Check for Students’ Understanding

1. Graph the given coordinates to transfer the star onto the x-y coordinate plane.
   
   \((0, 11), (2, 3), (9, 3), (4, -1), (5, -9), (0, -3), (-5, -9), (-4, -1), (-9, 3), (-2, 3)\)

2. Connect the coordinates to form the star and also sketch the circle and the heart on the star.

3. Without graphing, if the star was reflected over the line \(x = 12\), what are the new coordinates of the vertex of the star closest to the dot?
   \((15, 3)\)

4. Without graphing, if the star was reflected over the line \(x = 12\), what are the new coordinates of the vertex of the star closest to the heart?
   \((19, -9)\)
5. Without graphing, if the star was reflected over the line $y = 12$, what are the new coordinates of the vertex of the star closest to the dot?
   
   $(21, 3)$

6. Without graphing, if the star was reflected over the line $y = 12$, what are the new coordinates of the vertex of the star closest to the heart?
   
   $(5, 33)$
Essential Ideas

- Triangles are translated, rotated, and reflected in a coordinate plane.
- A translation is a transformation that slides each point of a figure the same distance and direction. The new figure created from the translation is called the image.
- A rotation is a transformation that turns a figure clockwise or counterclockwise about a fixed point for a given angle, and a given direction.
- A reflection is a transformation that flips a figure over a reflection line.

Texas Essential Knowledge and Skills for Mathematics

Grade 8

(10) Two-dimensional shapes. The student applies mathematical process standards to develop transformational geometry concepts. The student is expected to:

(B) differentiate between transformations that preserve congruence and those that do not

(C) explain the effect of translations, reflections over the $x$- or $y$-axis, and rotations limited to $90^\circ$, $180^\circ$, $270^\circ$, and $360^\circ$ as applied to two-dimensional shapes on a coordinate plane using an algebraic representation.
Overview

Students explore, compare, and generalize the characteristics of triangles that are translated, rotated, and reflected in a coordinate plane. They will determine the coordinates of an image that has been translated, rotated, or reflected.
Warm Up

1. What do a translation, a rotation, and a reflection have in common?

   A translation, a rotation, and a reflection are all transformations. Performing any of these transformations changes the location of a point or figure on the $x$-$y$ coordinate plane.

   All three transformations preserve the shape and the size of the figure.

2. What is the difference between a translation, a rotation, and a reflection?

   A translation is a transformation that slides each point of the figure the same distance and direction. The coordinates of the image are a result of the number of units and direction the pre-image is moved.

   A rotation is a transformation that turns a figure clockwise or counterclockwise about a fixed point for a given angle, and a given direction. The coordinates of the image are a result of the degrees of rotation, the direction of rotation, and the point of rotation.

   A reflection is a transformation that flips a figure over a reflection line. The coordinates of the image are a result of axis or line of reflection.
Learning Goals
In this lesson, you will:
- Translate triangles on a coordinate plane.
- Rotate triangles on a coordinate plane.
- Reflect triangles on a coordinate plane.

When you look at the night sky, you see bright stars and dim stars. But are the dimmer stars farther away from us or just less bright? Astronomers use a variety of methods to measure the universe, but at the end of the 1980s, they made vast improvements in the accuracy of these measurements.

In 1989, the Hipparcos satellite was launched by the European Space Agency. Among other advantages, this satellite was not affected by Earth's atmosphere and could view the entire “sky,” so it could provide more accurate measurements of distances. In 1997, the Hipparcos Catalogue was published, which contained high-precision distance information for more than 100,000 stars!
Problem 1
A point with the coordinates \((x, y)\) is located in the first quadrant. Students perform translations and record the coordinates of the images in terms of \(x\) and \(y\). They will generalize by describing the translation in terms of \(x\) and \(y\) that would move the point into each of the quadrants. Students are then given the coordinates of three vertices of a triangle and graph the triangle. Using translations, they will form two different triangles and record the coordinates of the vertices of the images. Finally, students are given the coordinates of the vertices of a triangle and without graphing they will determine the coordinates of images resulting from different translations.

ELL Tip
Provide students with a graphic organizer about the coordinate plane. Work with students to label the four quadrants, as well as the possible values of each coordinate written in algebraic format. For example, \((x, y), (-x, -y), (-x, y)\) and \((x, -y)\). Support students in plotting coordinates in each quadrant.

Problem 1 Translating Triangles on a Coordinate Plane
You have studied translations, rotations, and reflections of various geometric figures. In this lesson, you will explore, compare, and generalize the characteristics of triangles as you translate, rotate, and reflect them on a coordinate plane.

Consider the point \((x, y)\) located anywhere in the first quadrant of the coordinate plane.

1. Translate the point \((x, y)\) according to the descriptions in the table shown. Plot the point, and then record the coordinates of the translated points in terms of \(x\) and \(y\).

<table>
<thead>
<tr>
<th>Translation</th>
<th>Point ((x, y)) located in Q1</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 units to the left</td>
<td>((x - 3, y))</td>
</tr>
<tr>
<td>3 units down</td>
<td>((x, y - 3))</td>
</tr>
<tr>
<td>3 units to the right</td>
<td>((x + 3, y))</td>
</tr>
<tr>
<td>3 units up</td>
<td>((x, y + 3))</td>
</tr>
</tbody>
</table>

Grouping
- Ask a student to read the introduction before Question 1 aloud. Then discuss the information as a class.
- Have students complete Questions 1 and 2 with a partner. Then share the responses as a class.
Share Phase, Question 1
• Which quadrant of the x-y plane is the location of the point (x, y)?
• Which quadrant of the x-y plane is the location of the point (−x, y)?
• Which quadrant of the x-y plane is the location of the point (x, −y)?
• Which quadrant of the x-y plane is the location of the point (−x, −y)?

Share Phase, Question 2
• If a point with the coordinates (x, y) is translated −x units, what are the coordinates of its image?
• If a point with the coordinates (x, y) is translated −y units, what are the coordinates of its image?
• If a point with the coordinates (x, y) is translated less than −x units, how would you describe the coordinates of its image?
• If a point with the coordinates (x, y) is translated less than −y units, how would you describe the coordinates of its image?

Grouping
Have students complete Question 3 with a partner. Then share the responses as a class.

Share Phase, Question 3
• How did you determine the coordinates of the vertices of triangle ABC when it was translated right 5 units?
• How did you determine the coordinates of the vertices of triangle ABC when it was translated down 8 units?
• Could you have determined the coordinates of the vertices of the image without graphing? Explain.
Grouping
Have students complete Question 4 with a partner. Then share the responses as a class.

Share Phase, Question 4

- How did you determine the coordinates of point D’?
- How did you determine the coordinates of point E’?
- How did you determine the coordinates of point F’?
- How did you determine the coordinates of point D”?
- How did you determine the coordinates of point E”?
- How did you determine the coordinates of point F”?

Let’s consider the vertices of a different triangle and translations without graphing.

4. The vertices of triangle DEF are D(−7, 10), E(−5, 5), and F(−8, 1).
   a. If triangle DEF is translated to the right 12 units, what are the coordinates of the vertices of the image? Name the triangle.
      The coordinates of the vertices of the triangle D’E’F’ are D’(5, 10), E’(7, 5), and F’(4, 1).
   b. How did you determine the coordinates of the image without graphing the triangle?
      The coordinates of the vertices of the image were determined by adding 12 to each of the x-coordinates. The y-coordinates stayed the same.
   c. If triangle DEF is translated up 9 units, what are the coordinates of the vertices of the image? Name the triangle.
      The coordinates of the vertices of the triangle D”E”F” are D”(−7, 19), E”(−5, 14), F”(−8, 10).
   d. How did you determine the coordinates of the image without graphing the triangle?
      The coordinates of the vertices of the image were determined by adding 9 to each of the y-coordinates. The x-coordinates stayed the same.
**Problem 2**
A point with the coordinates \((x, y)\) is located in the first quadrant. Students perform rotations of 90° and 180° counterclockwise using the origin as the point of rotation and record the coordinates of the images in terms of \(x\) and \(y\). Next, they are given the coordinates of three vertices of a triangle and will graph the triangle. Using rotations of 90° and 180° counterclockwise and the origin as the point of rotation, students form two different triangles and record the coordinates of the vertices of the images. Finally, they are given the coordinates of a triangle and without graphing they will determine the coordinates of images resulting from different rotations.

**Grouping**
Have students complete Question 1 with a partner. Then share the responses as a class.

**Share Phase, Question 1**
- Which quadrant of the \(x\)-\(y\) plane is the location of the point \((x, y)\)?
- If a point with the coordinates \((x, y)\) is rotated about the origin 90° counterclockwise, in which quadrant does the image appear?

- If a point with the coordinates \((x, y)\) is rotated about the origin 180° counterclockwise, in which quadrant does the image appear?
- How many degrees rotation about the origin would be necessary for the image of the point with the coordinates \((x, y)\) to appear in the fourth quadrant? Explain.
Grouping
Have students complete Questions 2 and 3 with a partner. Then share the responses as a class.

Share Phase, Question 2
- How did you determine the coordinates of the vertices of the image of triangle ABC when the triangle was rotated about the origin 90° counter-clockwise?
- How did you determine the coordinates of the vertices of the image of triangle ABC when the triangle was rotated about the origin 180° counter-clockwise?
- Could you have determined the coordinates of the vertices of the image without graphing? Explain.

2. Graph triangle ABC by plotting the points A(3, 4), B(6, 1), and C(4, 9).

Use the table to record the coordinates of the vertices of each triangle.

<table>
<thead>
<tr>
<th>Original Triangle</th>
<th>Rotation About the Origin 90° Counterclockwise</th>
<th>Rotation About the Origin 180° Counterclockwise</th>
</tr>
</thead>
<tbody>
<tr>
<td>△ABC</td>
<td>△A'B'C'</td>
<td>△A''B''C''</td>
</tr>
<tr>
<td>A (3, 4)</td>
<td>A' (−4, 3)</td>
<td>A'' (−3, −4)</td>
</tr>
<tr>
<td>B (6, 1)</td>
<td>B' (−1, 6)</td>
<td>B'' (−6, −1)</td>
</tr>
<tr>
<td>C (4, 9)</td>
<td>C' (−9, 4)</td>
<td>C'' (−4, −9)</td>
</tr>
</tbody>
</table>
Share Phase, Question 3

- How did you determine the coordinates of point $D'$?
- How did you determine the coordinates of point $E'$?
- How did you determine the coordinates of point $F'$?
- How did you determine the coordinates of point $D''$?
- How did you determine the coordinates of point $E''$?
- How did you determine the coordinates of point $F''$?

Let's consider a different triangle and rotations without graphing.

3. The vertices of triangle $DEF$ are $D(-7, 10)$, $E(-5, 5)$, and $F(-1, -8)$.
   a. If triangle $DEF$ is rotated 90° counterclockwise, what are the coordinates of the vertices of the image? Name the rotated triangle. Then determine the coordinates of the vertices of the image.
      The coordinates of the vertices of triangle $D'F'E'$ are $D'(-10, -7)$, $E'(-5, -5)$, and $F'(8, -1)$.
   b. How did you determine the coordinates of the image without graphing the triangle?
      The coordinates of the vertices of the image were determined by switching the $x$- and $y$-coordinates while writing the opposite of the $y$-coordinate.
   c. If triangle $DEF$ is rotated 180° counterclockwise, what are the coordinates of the vertices of the image? Name the rotated triangle.
      The coordinates of the vertices of triangle $D''F'E''$ are $D''(7, -10)$, $E''(5, -5)$, and $F''(1, 8)$.
   d. How did you determine the coordinates of the image without graphing the triangle?
      The coordinates of the vertices of the image were determined by writing the opposite of each of the $x$-coordinates and the opposite of each of the $y$-coordinates.
Problem 3

A point with the coordinates \((x, y)\) is located in the first quadrant. Students reflect the point over the \(x\)-axis and the \(y\)-axis and record the coordinates of the images in terms of \(x\) and \(y\). Next, they are given the coordinates of three vertices of a triangle and will graph the triangle. Using reflections over the \(x\)-axis and \(y\)-axis, students form two different triangles and record the coordinates of the vertices of the images. Finally, they are given the coordinates of the vertices of a triangle and without graphing they will determine the coordinates of images resulting from different reflections.

Grouping

Have students complete Question 1 with a partner. Then share the responses as a class.

Share Phase, Question 1

- Which quadrant of the \(x\)-\(y\) plane is the location of the point \((x, y)\)?
- If a point with the coordinates \((x, y)\) is reflected over the \(x\)-axis, in which quadrant does the image appear?
- If a point with the coordinates \((x, y)\) is reflected over the \(y\)-axis, in which quadrant does the image appear?
- What type of reflection would be necessary for the image of the point \((x, y)\) to appear in the third quadrant? Explain.
Grouping
Have students complete Questions 2 and 3 with a partner. Then share the responses as a class.

Share Phase, Question 2
- How did you determine the coordinates of the vertices of the image of triangle ABC when the triangle was reflected over the x-axis?
- How did you determine the coordinates of the vertices of the image of triangle ABC when the triangle was reflected over the y-axis?
- Could you have determined the coordinates of the vertices of the image without graphing? Explain.

2. Graph triangle ABC by plotting the points A(3, 4), B(6, 1), and C(4, 9).

Use the table to record the coordinates of the vertices of each triangle.

<table>
<thead>
<tr>
<th>Original Triangle</th>
<th>Triangle Reflected Across the x-axis</th>
<th>Triangle Reflected Across the y-axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>△ABC</td>
<td>△A'B'C'</td>
<td>△A''B''C''</td>
</tr>
<tr>
<td>A (3, 4)</td>
<td>A' (3, -4)</td>
<td>A'' (-3, 4)</td>
</tr>
<tr>
<td>B (6, 1)</td>
<td>B' (6, -1)</td>
<td>B'' (-6, 1)</td>
</tr>
<tr>
<td>C (4, 9)</td>
<td>C' (4, -9)</td>
<td>C'' (-4, 9)</td>
</tr>
</tbody>
</table>

Do you see any patterns?
Share Phase,
Question 3

• How did you determine the coordinates of point D’?
• How did you determine the coordinates of point E’?
• How did you determine the coordinates of point F’?
• How did you determine the coordinates of point D”?
• How did you determine the coordinates of point E”?
• How did you determine the coordinates of point F”?

Let’s consider a different triangle and reflections without graphing.

3. The vertices of triangle DEF are D(−7, 10), E(−5, 5), and F(−1, −8).

   a. If triangle DEF is reflected across the x-axis, what are the coordinates of the vertices of the image? Name the triangle.
      The coordinates of the vertices of triangle D’E’F’ are D’(−7, −10), E’(−5, −5), and F’(−1, 8).

   b. How did you determine the coordinates of the image without graphing the triangle?
      The coordinates of the vertices of triangle D’E’F’ were determined by writing the opposite of each of the y-coordinates. The x-coordinates stayed the same.

   c. If triangle DEF is reflected across the y-axis, what are the coordinates of the vertices of the image? Name the triangle.
      The coordinates of the vertices of triangle D”E”F” are D”(7, 10), E”(5, 5), and F”(1, −8).

   d. How did you determine the coordinates of the image without graphing the triangle?
      The coordinates of the vertices of triangle D”E”F” were determined by writing the opposite of each of the x-coordinates. The y-coordinates stayed the same.
**Talk the Talk**

Students are given the coordinates of a pre-image and the coordinates of its image. From this information they will describe the transformation used to form the image and explain their reasoning. The transformations include translations, rotations, and reflections.

**Grouping**

Have students complete Questions 1 through 4 with a partner. Then share the responses as a class.

---

1. The vertices of triangle $PQR$ are $P(4,3), Q(2,2),$ and $R(0,0)$. Describe the translation used to form each triangle. Explain your reasoning.
   - **a.** $P'(0,3), Q'(-6,2),$ and $R'(-4,0)$
     
     Triangle $P'Q'R'$ was formed by translating triangle $PQR$ 4 units to the left. Each $x$-value of the translated triangle is 4 less than the corresponding $x$-value of the original triangle.
   - **b.** $P'(4,5.5), Q'(-2,4.5),$ and $R'(0,2.5)$
     
     Triangle $P'Q'R'$ was formed by translating triangle $PQR$ up 2.5 units. Each $y$-value of the translated triangle is 2.5 more than the corresponding $y$-value of the original triangle.

2. The vertices of triangle $JME$ are $J(1,3), M(6,5),$ and $E(8,1)$. Describe the rotation used to form each triangle. Explain your reasoning.
   - **a.** $J'(-3,1), M'(-5,6),$ and $E'(-1,8)$
     
     Triangle $J'M'E'$ was formed by rotating triangle $JME$ $90^\circ$ counterclockwise about the origin. Each ordered pair of triangle $J'M'E'$ is $(-y,x)$ in comparison to the ordered pairs of triangle $JME$.
   - **b.** $J'(-1,-3), M'(-6,-5),$ and $E'(-8,-1)$
     
     Triangle $J'M'E'$ was formed by rotating triangle $JME$ $180^\circ$ counterclockwise about the origin. Each ordered pair of triangle $J'M'E'$ is $(-x,-y)$ in comparison to the ordered pairs of triangle $JME$. 

---
Share Phase, Questions 1 through 4

- What strategies did you use to determine the transformations for each question?
- Why are the images congruent to the original figures? Explain.

3. The vertices of triangle NRT are N(12, 4), R(14, 1), and T(20, 9). Describe the reflection used to form each triangle. Explain your reasoning.
   a. N'(−12, 4), R'(−14, 1), and T'(−20, 9)
      Triangle N'R'T' was formed by reflecting triangle NRT across the y-axis. Each ordered pair of triangle N'R'T' is (−x, y) in comparison to the ordered pairs of triangle NRT.

   b. N''(12, −4), R''(14, −1), and T''(20, −9)
      Triangle N''R''T'' was formed by reflecting triangle NRT across the x-axis. Each ordered pair of triangle N''R''T'' is (x, −y) in comparison to the ordered pairs of triangle NRT.

4. Are all the images that result from a translation, rotation, or reflection (always, sometimes, or never) congruent to the original figure?
   All images that result from a translation, rotation, or reflection are always congruent to the original figure.

Be prepared to share your solutions and methods.
Follow Up

Assignment
Use the Assignment for Lesson 3.4 in the Student Assignments book. See the Teacher’s Resources and Assessments book for answers.

Skills Practice
Refer to the Skills Practice worksheet for Lesson 3.4 in the Student Assignments book for additional resources. See the Teacher’s Resources and Assessments book for answers.

Assessment
See the Assessments provided in the Teacher’s Resources and Assessments book for Chapter 3.

Check for Students’ Understanding
The coordinates of a pre-image are given.

A (0, 0)  B (13, 0)  C (13, 4)  D (4, 4)

Consider the coordinates of each image listed and describe the transformation.

1. A’ (0, −7)  B’ (13, −7)  C’ (13, −3)  D’ (4, −3)
   A translation down 7 units

2. A’ (0, 0)  B’ (−13, 0)  C’ (−13, 4)  D’ (−4, 4)
   A reflection over the y-axis

3. A’ (0, 0)  B’ (−13, 0)  C’ (−13, −4)  D’ (−4, −4)
   A rotation 180° counterclockwise about the origin

4. Did you have to graph the pre-image and image to describe the transformation? Explain your reasoning.
   Answers will vary.
   No. I did not have to graph the pre-image and image to describe the transformation. I was able to recognize the change in the coordinates of the pre-image to the image and knew what transformation the change in values implied.
### Essential Ideas
- Congruent line segments are line segments that have the same length.
- Congruent angles are angles that are equal in measure.
- Corresponding sides are sides that have the same relative positions in geometric figures.
- Corresponding angles are angles that have the same relative position in geometric figures.

### Key Terms
- congruent line segments
- congruent angles
- corresponding sides
- corresponding angles

### Learning Goals
- Identify corresponding sides and corresponding angles of congruent triangles.
- Explore the relationship between corresponding sides of congruent triangles.
- Explore the relationship between corresponding angles of congruent triangles.
- Write statements of triangle congruence.
- Identify and use transformations to create new images.

### Texas Essential Knowledge and Skills for Mathematics
**Grade 8**

(10) Two-dimensional shapes. The student applies mathematical process standards to develop transformational geometry concepts. The student is expected to:

(A) generalize the properties of orientation and congruence of rotations, reflections, translations, and dilations of two-dimensional shapes on a coordinate plane

(B) differentiate between transformations that preserve congruence and those that do not
Overview
The terms congruent line segments, congruent angles, corresponding sides, and corresponding angles are defined. Students explore the properties of congruent triangles on a coordinate plane. Students will use a translation and the Pythagorean Theorem to determine corresponding sides of congruent triangles are congruent. They also use a protractor to determine the corresponding angles of congruent triangles are congruent. Finally, students will write triangle congruence statements and use the statements to list congruent corresponding sides and congruent corresponding angles.
Warm Up

1. Determine the length of each side of triangle $NTH$.
   
   $NT = 3.6$ cm
   $TH = 10$ cm
   $NH = 12$ cm

2. Determine the measure of each angle in triangle $NTH$.
   
   $m\angle N = 47^\circ$
   $m\angle T = 117^\circ$
   $m\angle H = 16^\circ$

3. If triangle $NTH$ is translated down 9 units, how would the length of the sides of the image compare to the length of the sides of the pre-image?
   
   They would be the same.

4. If triangle $NTH$ is rotated $90^\circ$ counterclockwise about the origin, how would the length of the sides of the image compare to the length of the sides of the pre-image?
   
   They would be the same.

5. If triangle $NTH$ is reflected over the $x$-axis, how would the length of the sides of the image compare to the length of the sides of the pre-image?
   
   They would be the same.

6. If triangle $NTH$ is translated up 3 units, how would the measures of the angles of the image compare to the measures of the angles of the pre-image?
   
   They would be the same.
7. If triangle $NTH$ is rotated $180^\circ$ counterclockwise about the origin, how would the measures of the angles of the image compare to the measures of the angles of the pre-image?
   They would be the same.

8. If triangle $NTH$ is reflected over the $y$-axis, how would the measures of the angles of the image compare to the measures of the angles of the pre-image?
   They would be the same.
In mathematics, when a geometric figure is translated, reflected, or rotated, the size and shape of the figure doesn't change. But in physics, things are a little different. An idea called length contraction in physics means that when an object is in motion, its length appears to be slightly less than it really is. The faster the object is moving, the smaller it appears. If an object is moving at the speed of light, it would be practically invisible!
**Problem 1**

Congruent line segments and congruent angles are defined. Symbols are used to represent side and angle relationships. A distinction is made between the symbol used to represent the actual length of a line segment and the symbol used to represent the geometric model of a line segment. A distinction is also made between the symbol used to represent the measure of an angle and the symbol used to represent the geometric model of an angle.

In the previous lesson, you determined that if a triangle was translated, rotated, or reflected, it resulted in creating an image that was the same size and the same shape as the original triangle; therefore, the image and the original triangle are said to be congruent triangles.

**Congruent line segments** are line segments that have the same length. Congruent triangles are triangles that are the same size and the same shape.

If the length of line segment $AB$ is equal to the length of line segment $DE$, the relationship can be expressed using symbols. These are a few examples.

- $AB = DE$ is read “the distance between $A$ and $B$ is equal to the distance between $D$ and $E$”
- $m\overline{AB} = m\overline{DE}$ is read “the measure of line segment $AB$ is equal to the measure of line segment $DE$.”

If the sides of two different triangles are equal in length, for example, the length of side $AB$ in triangle $ABC$ is equal to the length of side $DE$ in triangle $DEF$, these sides are said to be congruent. This relationship can be expressed using symbols.

- $\overline{AB} \cong \overline{DE}$ is read “line segment $AB$ is congruent to line segment $DE$.”

**Congruent angles** are angles that are equal in measure.

If the measure of angle $A$ is equal to the measure of angle $D$, the relationship can be expressed using symbols.

- $m\angle A = m\angle D$ is read “the measure of angle $A$ is equal to the measure of angle $D$.”

If the angles of two different triangles are equal in measure, for example, the measure of angle $A$ in triangle $ABC$ is equal to the measure of angle $D$ in triangle $DEF$, these angles are said to be congruent. This relationship can be expressed using symbols.

- $\angle A \cong \angle D$ is read “angle $A$ is congruent to angle $D$.”

Discuss Phase, Problem 1

- What is the difference between the length of a line segment and a line segment?
- Are line segments said to be congruent or equal?
- Are the lengths of line segments said to be congruent or equal?
- Are the measures of line segments said to be congruent or equal?
- What is the difference between the measure of an angle and an angle?
- Are angles said to be congruent or equal?
- Are measures of angles said to be congruent or equal?

Instruct students to read and take notes on congruent line segments and congruent angles. Provide students with a concept map to support this stage in their note taking.
**Problem 2**
The definition of corresponding sides is provided. Students are given the coordinates of the vertices of a right triangle and will graph the triangle. After determining the length of each side using the Pythagorean Theorem and the distance between two horizontal and vertical points, students will translate the triangle. They then determine the lengths of the sides of the image and conclude that corresponding sides are congruent.

**Grouping**
- Have students complete Question 1 with a partner. Then share the responses as a class.
- Have students complete Questions 2 through 7 with a partner. Then share the responses as a class.

**Share Phase, Question 1**
- Triangle ABC lies in which quadrant?
- Is triangle ABC a right triangle? How do you know?
- How did you determine the length of the legs of the right triangle?
- What do you know about the hypotenuse of a right triangle?
- What do you need to know to determine the length of the hypotenuse?

**Problem 2**
**Corresponding Sides of Congruent Triangles**

Let’s explore the properties of congruent triangles.

1. Graph triangle ABC by plotting the points A(8, 10), B(1, 2), and C(8, 2).

   ![Graph of triangle ABC](image)

   - a. Describe triangle ABC.
     - Triangle ABC is a right triangle.

   - b. Use the coordinate plane to determine the lengths of sides AC and BC.
     - The length of side AC is 8 units.
     - The length of side BC is 7 units.

   - c. Use the Pythagorean Theorem to determine the length of side AB.
     \[ a^2 + b^2 = c^2 \]
     \[ 8^2 + 7^2 = c^2 \]
     \[ 64 + 49 = c^2 \]
     \[ c^2 = 113 \]
     \[ c = \sqrt{113} \approx 10.6 \]
     - The length of side AB is approximately 10.6 units.

2. Translate triangle ABC 10 units to the left to form triangle DEF. Graph triangle DEF and list the coordinates of points D, E, and F.
   - The coordinates of triangle DEF are D(−2, 10), E(−9, 2) and F(−2, 2).
Share Phase, Questions 2 through 7

- How did you determine the coordinates of the vertices of the image of triangle ABC?
- Which side of triangle ABC is in the same relative position as side DE in triangle DEF?
- Which side of triangle ABC is in the same relative position as side DF in triangle DEF?
- Did you have to use the Pythagorean Theorem to determine the length of side DE? Why or why not?
- Do you think corresponding sides of any congruent triangles are congruent, or just corresponding sides of right triangles? Explain.

Corresponding sides are sides that have the same relative positions in geometric figures. Triangle ABC and triangle DEF in Question 1 are the same size and the same shape. Each side in triangle ABC matches or corresponds to a specific side in triangle DEF.

3. What would you predict to be true about the lengths of corresponding sides of congruent triangles?
   The lengths of corresponding sides of congruent triangles are equal.

4. Identify the pairs of corresponding sides of triangle ABC and triangle DEF.
   a. Side AC in triangle ABC corresponds to what side in triangle DEF?
      Side AC in triangle ABC corresponds to side DF in triangle DEF.
   b. Side BC in triangle ABC corresponds to what side in triangle DEF?
      Side BC in triangle ABC corresponds to side EF in triangle DEF.
   c. Side AB in triangle ABC corresponds to what side in triangle DEF?
      Side AB in triangle ABC corresponds to side DE in triangle DEF.

5. Determine the side lengths of triangle DEF.
   a. mDF
      The length of side DF is 8 units.
   b. mEF
      The length of side EF is 7 units.
   c. mDE
      The length of side DE is approximately 10.6 units.

6. Compare the lengths of the sides in triangle ABC to the lengths of the corresponding sides in triangle DEF.
   a. How does the length of side AC compare to the length of side DF?
      The length of side AC is equal to the length of side DF.
   b. How does the length of side BC compare to the length of side EF?
      The length of side BC is equal to the length of side EF.
   c. How does the length of side AB compare to the length of side DE?
      The length of side AB is equal to the length of side DE.

7. In general, what can be said about the relationship between the corresponding sides of congruent triangles?
   Corresponding sides of congruent triangles are congruent.
Problem 3
Students use triangles ABC and DEF from Problem 2 in conjunction with a protractor to determine the corresponding angles of congruent triangles are indeed congruent.

Materials
Protractor

Grouping
Have students complete Questions 1 through 6 with a partner. Then share the responses as a class.

Share Phase, Questions 1 through 6

- How did you know which scale to use on the protractor when measuring the angles of triangle ABC?
- Which angle of triangle ABC is in the same relative position as ∠D in triangle DEF?
- Which angle of triangle ABC is in the same relative position as ∠E in triangle DEF?
- Which angle of triangle ABC is in the same relative position as ∠F in triangle DEF?
- Do you think corresponding angles of any congruent triangles are congruent, or just corresponding angles of right triangles? Explain.

Triangle ABC and triangle DEF are the same size and the same shape. Each angle in triangle ABC matches, or corresponds, to a specific angle in triangle DEF. Corresponding angles are angles that have the same relative positions in geometric figures.

2. What would you predict to be true about the measures of corresponding angles of congruent triangles?
   The measures of corresponding angles of congruent triangles are equal.

3. Identify the corresponding angles of triangle ABC and triangle DEF.
   a. Angle A in triangle ABC corresponds to what angle in triangle DEF?
      Angle A in triangle ABC corresponds to angle D in triangle DEF.
   b. Angle B in triangle ABC corresponds to what angle in triangle DEF?
      Angle B in triangle ABC corresponds to angle E in triangle DEF.
   c. Angle C in triangle ABC corresponds to what angle in triangle DEF?
      Angle C in triangle ABC corresponds to angle F in triangle DEF.

4. Use a protractor to determine the measures of angles D, E, and F.
   The measure of angle F is equal to 90°.
   The measure of angle D is equal to 40°.
   The measure of angle E is equal to 50°.

Problem 3  Corresponding Angles of Congruent Triangles

Use triangle ABC and triangle DEF in Problem 2 to answer each question.

1. Use a protractor to determine the measure of angles A, B, and C.
   The measure of angle C is equal to 90°.
   The measure of angle A is equal to 40°.
   The measure of angle B is equal to 50°.
Problem 4
Students practice listing congruent corresponding sides and angles of two triangles given only a congruence statement. An image and pre-image of a triangle is provided and students will identify the transformation used, acknowledge the transformation preserves both size and shape, write a triangle congruence statement, and use the statement to list the congruent sides and congruent angles.

Grouping
Ask a student to read Question 1 aloud. Discuss and complete Question 1 as a class.

Discuss Phase, Question 1
• Do you need a diagram of the triangles to determine the congruent corresponding sides? Why or why not?
• Do you need a diagram of the triangles to determine the congruent corresponding angles? Why or why not?
• How is the triangle congruency statement used to determine the congruent corresponding sides?
• How is the triangle congruency statement used to determine the congruent corresponding angles?

Problem 4 Statements of Triangle Congruence

1. Consider the congruence statement \( \triangle JRB \cong \triangle MNS \).
   a. Identify the congruent angles.
      \( \angle J = \angle M \)
      \( \angle R = \angle N \)
      \( \angle B = \angle S \)
   b. Identify the congruent sides.
      \( JR = MN \)
      \( RB = NS \)
      \( JB = MS \)

So, if you know that two angles are congruent then you also know their measures are equal.

5. Compare the measures of the angles in triangle \( ABC \) to the measures of the corresponding angles in triangle \( DEF \).
   a. How does the measure of angle \( A \) compare to the measure of angle \( D \)?
      The measure of angle \( A \) is equal to the measure of angle \( D \).
   b. How does the measure of angle \( B \) compare to the measure of angle \( E \)?
      The measure of angle \( B \) is equal to the measure of angle \( E \).
   c. How does the measure of angle \( C \) compare to the measure of angle \( F \)?
      The measure of angle \( C \) is equal to the measure of angle \( F \).

6. In general, what can be said about the relationship between the corresponding angles of congruent triangles?
   Corresponding angles of congruent triangles are congruent.
Grouping
Have students complete Questions 2 and 3 with a partner. Then share the responses as a class.

Share Phase, Question 2
• How could you tell the transformation was not a translation?
• How could you tell the transformation was not a reflection?
• Do all of the transformations you have studied preserve both size and shape? Explain.
• Is there more than one way to write the triangle congruence statement?
• If the triangle congruence statement is written differently, will that change the congruent parts of the triangle? Explain.

2. Analyze the two triangles shown.

a. Identify the transformation used to create triangle PMK. Triangle TWC was rotated to create triangle PMK.

b. Does the transformation used preserve the size and shape of the triangle? Yes. Rotations preserve both the size and shape of figures.

c. Using the triangles shown, write a triangle congruence statement. \( \triangle TWC \cong \triangle PMK \)

d. Using your congruence statement, identify the congruent angles. 
   \( \angle T = \angle P \)
   \( \angle W = \angle M \)
   \( \angle C = \angle K \)

e. Using your congruence statement, identify the congruent sides. 
   \( TW = PM \)
   \( WC = MK \)
   \( TC = PK \)
Share Phase, Question 3

- In Question 3, how could you tell the transformation was not a translation?
- In Question 3, how could you tell the transformation was not a rotation?
- What is another way to write the triangle congruence statement?

3. Analyze the two triangles shown.

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6</td>
<td>0</td>
</tr>
<tr>
<td>-10</td>
<td>-8</td>
</tr>
</tbody>
</table>

a. Identify the transformation used to create triangle ZQV.

Triangle TRG was reflected across the x-axis to create triangle ZQV.

b. Does the transformation used preserve the size and shape of the triangle?

Yes. Reflections preserve both the size and shape of figures.

c. Using the triangles shown, write a triangle congruence statement.

\[ \triangle TRG \cong \triangle ZQV \]

d. Using your congruence statement, identify the congruent angles.

\[ \angle T = \angle Z \]
\[ \angle R = \angle Q \]
\[ \angle G = \angle V \]

e. Using your congruence statement, identify the congruent sides.

\[ TR = ZQ \]
\[ RG = QV \]
\[ TG = ZV \]
**Talk the Talk**

Students describe the characteristics of congruent triangles.

**Grouping**

Have students complete Questions 1 and 2 with a partner. Then share the responses as a class.

---

1. Given any triangle on a coordinate plane, how can you create a different triangle that you know will be congruent to the original triangle?

   I can translate, rotate, or reflect any triangle to create a congruent triangle.

2. Describe the properties of congruent triangles.

   Congruent triangles have corresponding angles and corresponding side lengths that have equal measures.

Be prepared to share your solutions and methods.
Follow Up

Assignment
Use the Assignment for Lesson 3.5 in the Student Assignments book. See the Teacher’s Resources and Assessments book for answers.

Skills Practice
Refer to the Skills Practice worksheet for Lesson 3.5 in the Student Assignments book for additional resources. See the Teacher’s Resources and Assessments book for answers.

Assessment
See the Assessments provided in the Teacher’s Resources and Assessments book for Chapter 3.

Check for Students’ Understanding
1. Given:
   \( \angle B \cong \angle K \)
   \( \angle W \cong \angle M \)
   \( \angle P \cong \angle C \)

   Write a triangle congruency statement based on the congruent corresponding angles given.
   \( \triangle BWP \cong \triangle KMC \)

2. Given:
   \( \overline{OV} \cong \overline{SR} \)
   \( \overline{VT} \cong \overline{RX} \)
   \( \overline{OT} \cong \overline{SX} \)

   Write a triangle congruency statement based on the congruent corresponding sides given.
   \( \triangle OVT \cong \triangle SRX \)

3. Given:
   \( \angle H \cong \angle Z \)
   \( \overline{HM} \cong \overline{ZG} \)
   \( \angle Y \cong \angle D \)

   Write a triangle congruency statement based on the congruent corresponding angles and congruent corresponding sides given.
   \( \triangle YHM \cong \triangle DZG \)
3.1 Translating Geometric Figures

A translation is a transformation that “slides” each point of a figure the same distance and direction. Sliding a figure left or right is a horizontal translation and sliding it up or down is a vertical translation. The new figure created from a translation is called the image.

Example

\( \triangle ABC \) with coordinates \( A(-2, 2), B(0, 5), \) and \( C(1, 1) \) is translated six units horizontally and -4 units vertically.

The coordinates of the image are \( A'(4, -2), B'(6, 1), \) and \( C'(7, -3). \)
Rotating Geometric Figures on a Coordinate Plane

A rotation is a transformation that turns a figure about a fixed point for a given angle and a given direction. The given angle is called the angle of rotation. The angle of rotation is the amount of rotation about a fixed point. The point around which the figure is rotated is called the point of rotation. Rotations can be either clockwise or counterclockwise.

Example

To rotate $\triangle XYZ$ 45° clockwise around point $Z$, use a protractor to draw a 45° angle as shown, with point $Z$ as the vertex. Next, rotate the figure clockwise around point $Z$ until the side corresponding to $YZ$ has been rotated 45°. The image is labeled as $\triangle X'Y'Z$. 
Reflecting Geometric Figures on the Coordinate Plane

A reflection is a transformation that “flips” a figure across a reflection line. A reflection line is a line that acts as a mirror such that corresponding points in the figure and its image are the same distance from the line.

When a figure is reflected across the x-axis, the y-values of the points on the image have the opposite sign of the y-values of the corresponding points on the original figure while the x-values remain the same. When a figure is reflected across the y-axis, the x-values of the points on the image have the opposite sign of the x-values of the corresponding points on the original figure while the y-values remain the same.

Example

A square with vertices $P(-1, 5)$, $Q(2, 8)$, $R(5, 5)$, and $S(2, 2)$ is reflected across the x-axis.

To determine the vertices of the image, change the sign of the y-coordinates of the figure’s vertices to find the y-coordinates of the image’s vertices. The x-coordinates remain the same. The vertices of the image are $P'(-1, -5)$, $Q'(2, -8)$, $R'(5, -5)$, and $S'(2, -2)$.
Translating Triangles in the Coordinate Plane

To translate a triangle in the coordinate plane means to move or “slide” the triangle to a new location without rotating it.

Example

Triangle $ABC$ has been translated 10 units to the left and 2 units down to create triangle $A'B'C'$.

The coordinates of triangle $ABC$ are $A(2, 8), B(7, 5)$, and $C(2, 5)$.

The coordinates of triangle $A'B'C'$ are $A'(−8, 6), B'(−3, 3)$, and $C'(−8, 3)$. 
To rotate a triangle in the coordinate plane means to “turn” the triangle either clockwise or counterclockwise about a fixed point, which is usually the origin. To determine the new coordinates of a point after a rotation, refer to the following table.

<table>
<thead>
<tr>
<th>Original Point</th>
<th>Rotation About the Origin 90° Counterclockwise</th>
<th>Rotation About the Origin 180° Counterclockwise</th>
<th>Rotation About the Origin 270° Counterclockwise</th>
<th>Rotation About the Origin 360° Counterclockwise</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(x, y)$</td>
<td>$(-y, x)$</td>
<td>$(-x, -y)$</td>
<td>$(y, -x)$</td>
<td>$(x, y)$</td>
</tr>
</tbody>
</table>

Example

Triangle $ABC$ has been rotated 180° counterclockwise about the origin to create triangle $A'B'C'$.

The coordinates of triangle $ABC$ are $A(2, 8)$, $B(7, 5)$, and $C(2, 5)$.

The coordinates of triangle $A'B'C'$ are $A'(-2, -8)$, $B'(-7, -5)$, and $C'(-2, -5)$. 

3.4 Rotating Triangles in the Coordinate Plane
3.4 Reflecting Triangles on a Coordinate Plane

To reflect a triangle on a coordinate plane means to “mirror” the triangle across a line of reflection to create a new triangle. Each point in the new triangle will be the same distance from the line of reflection as the corresponding point in the original triangle. To determine the coordinates of a point after a reflection across the $x$-axis, change the sign of the $y$-coordinate in the original point. To determine the coordinates of a point after a reflection across the $y$-axis, change the sign of the $x$-coordinate in the original point.

**Example**

Triangle $ABC$ has been reflected across the $x$-axis to create triangle $A'B'C'$.

The coordinates of triangle $ABC$ are $A(2, 8), B(7, 5), \text{ and } C(2, 5)$.

The coordinates of triangle $A'B'C'$ are $A'(2, -8), B'(7, -5), \text{ and } C'(2, -5)$. 
Identifying Corresponding Sides and Angles of Congruent Triangles

Congruent figures are figures that are the same size and the same shape. Congruent triangles are triangles that are the same size and the same shape. Congruent line segments are line segments that are equal in length. Congruent angles are angles that are equal in measure. In congruent figures, the corresponding angles are congruent and the corresponding sides are congruent.

Example

Triangle $DEF$ has been reflected across the $y$-axis to create triangle $PQR$.

- Line segment $DE$ corresponds to line segment $PQ$, which means $DE \equiv PQ$.
- Line segment $EF$ corresponds to line segment $QR$, which means $EF \equiv QR$.
- Line segment $DF$ corresponds to line segment $PR$, which means $DF \equiv PR$.
- Angle $D$ corresponds to angle $P$, which means $\angle D \equiv \angle P$.
- Angle $E$ corresponds to angle $Q$, which means $\angle E \equiv \angle Q$.
- Angle $F$ corresponds to angle $R$, which means $\angle F \equiv \angle R$.

The congruence statement for these two congruent triangles is $\triangle DEF \equiv \triangle PQR$. 

![Diagram of congruent triangles](image_url)