What is the distance from the Earth to the Moon? Don’t let drawings or even photos fool you. A lot of them can be misleading, making the Moon appear closer than it really is, which is about 250,000 miles away.

2.1 Soon You Will Determine the Right Triangle Connection
The Pythagorean Theorem......................................................... 45

2.2 Can That Be Right?
The Converse of the Pythagorean Theorem......................... 67

2.3 Pythagoras to the Rescue
Solving for Unknown Lengths.................................................... 75

2.4 Meeting Friends
The Distance Between Two Points
in a Coordinate System............................................................. 81

2.5 Diagonally
Diagonals in Two Dimensions .................................................. 89

2.6 Two Dimensions Meet Three Dimensions
Diagonals in Three Dimensions.................................................. 97
# Chapter 2 Overview

This chapter develops the Pythagorean Theorem and the Converse of the Pythagorean Theorem. These theorems are then applied to solve real-world problems in two and three dimensions.

<table>
<thead>
<tr>
<th>Lessons</th>
<th>TEKS</th>
<th>Pacing</th>
<th>Highlights</th>
<th>Models</th>
<th>Worked Examples</th>
<th>Peer Analysis</th>
<th>Talk the Talk</th>
<th>Technology</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1 The Pythagorean Theorem</td>
<td>8.6.C 8.7.C</td>
<td>1-2</td>
<td>This lesson presents three different activities for students to explore the relationship between the areas of the squares on each side of a right triangle, and then the Pythagorean Theorem is formally stated. Questions ask students to use the Pythagorean Theorem to solve real-world problems.</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>2.2 The Converse of the Pythagorean Theorem</td>
<td>8.7.C 8.7.D</td>
<td>1</td>
<td>This lesson states the Converse of the Pythagorean Theorem and explores Pythagorean triples. Questions ask students to apply the Pythagorean Theorem and the Converse of the Pythagorean Theorem to solve real-world problems.</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>2.3 Solving for Unknown Lengths</td>
<td>8.7.C</td>
<td>1</td>
<td>This lesson provides practice applying the Pythagorean Theorem and the Converse of the Pythagorean Theorem to solve for unknown side lengths and real-world problems.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>2.4 The Distance Between Two Points in a Coordinate System</td>
<td>8.7.D</td>
<td>1</td>
<td>This lesson extends the application of the Pythagorean Theorem to calculate the diagonal distance between two points on a coordinate plane.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>2.5 Diagonals in Two Dimensions</td>
<td>8.7.C 8.7.D</td>
<td>1</td>
<td>This lesson explores the relationship of the diagonal lengths of various geometric figures using the Pythagorean Theorem. Questions ask students to calculate the area of composite figures.</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>2.6 Diagonals in Three Dimensions</td>
<td>8.7.C</td>
<td>1</td>
<td>This lesson extends the application of the Pythagorean Theorem to determine the length of a three-dimensional diagonal of a rectangular solid.</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td>X</td>
</tr>
</tbody>
</table>
## Skills Practice Correlation for Chapter 2

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Problem Set</th>
<th>Objectives(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Vocabulary</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 - 6</td>
<td>Determine and verify the lengths of hypotenuses</td>
</tr>
<tr>
<td></td>
<td>7 - 12</td>
<td>Calculate lengths of hypotenuses for given triangles</td>
</tr>
<tr>
<td></td>
<td>13 - 16</td>
<td>Answer problem situations using the Pythagorean Theorem</td>
</tr>
<tr>
<td></td>
<td>17 - 24</td>
<td>Calculate lengths of unknown sides of triangles</td>
</tr>
<tr>
<td>2.2</td>
<td>Vocabulary</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 - 6</td>
<td>Determine if triangles are right triangles from given side lengths</td>
</tr>
<tr>
<td></td>
<td>7 - 12</td>
<td>Answer problem situations using the Pythagorean Theorem</td>
</tr>
<tr>
<td></td>
<td>13 - 18</td>
<td>Calculate lengths of line segments on coordinate planes</td>
</tr>
<tr>
<td>2.3</td>
<td>Vocabulary</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 - 6</td>
<td>Determine lengths of hypotenuses for given triangles</td>
</tr>
<tr>
<td></td>
<td>7 - 12</td>
<td>Determine unknown leg lengths</td>
</tr>
<tr>
<td></td>
<td>13 - 18</td>
<td>Use the Pythagorean Theorem to determine if given triangles are right triangles</td>
</tr>
<tr>
<td></td>
<td>19 - 24</td>
<td>Calculate unknown lengths using the Pythagorean Theorem</td>
</tr>
<tr>
<td>2.4</td>
<td>Vocabulary</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 - 8</td>
<td>Determine the distance between two points on coordinate planes by creating right triangles</td>
</tr>
<tr>
<td></td>
<td>9 - 16</td>
<td>Determine the distance between two objects on coordinate planes by creating right triangles</td>
</tr>
<tr>
<td>2.5</td>
<td>Vocabulary</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 - 6</td>
<td>Determine lengths of diagonals in given quadrilaterals</td>
</tr>
<tr>
<td></td>
<td>7 - 12</td>
<td>Calculate the area of shaded regions</td>
</tr>
<tr>
<td>2.6</td>
<td>Vocabulary</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 - 6</td>
<td>Draw all edges of rectangular solids then draw three-dimensional diagonals</td>
</tr>
<tr>
<td></td>
<td>7 - 12</td>
<td>Determine lengths of three-dimensional diagonals using the Pythagorean Theorem</td>
</tr>
<tr>
<td></td>
<td>13 - 18</td>
<td>Use given diagonals to determine the length of three-dimensional diagonals</td>
</tr>
<tr>
<td></td>
<td>19 - 24</td>
<td>Use the diagonal formula to answer questions</td>
</tr>
<tr>
<td></td>
<td>25 - 30</td>
<td>Determine unknown measurements in problem situations</td>
</tr>
</tbody>
</table>
Learning Goals
In this lesson, you will:
- Use mathematical properties to discover the Pythagorean Theorem.
- Solve problems involving right triangles.

Essential Ideas
- A right angle is an angle that measures 90° and a right triangle is a triangle with exactly one right angle.
- The leg of a right triangle is one of the two shorter sides and the hypotenuse is the side opposite the right angle.
- The Pythagorean Theorem states that if a and b are the lengths of the legs of a right triangle and c is the length of the hypotenuse, then $a^2 + b^2 = c^2$.
- A postulate is a mathematical statement that cannot be proven, but is considered true.
- A theorem is a mathematical statement that can be proven using definitions, postulates, and other theorems.
- A proof is a series of steps used to prove the validity of a statement.

Key Terms
- right triangle
- right angle
- leg
- hypotenuse
- diagonal of a square
- Pythagorean Theorem
- theorem
- postulate
- proof

Texas Essential Knowledge and Skills for Mathematics

Grade 8
(6) Expressions, equations, and relationships. The student applies mathematical process standards to develop mathematical relationships and make connections to geometric formulas. The student is expected to:

- (C) use models and diagrams to explain the Pythagorean theorem

(7) Expressions, equations, and relationships. The student applies mathematical process standards to use geometry to solve problems. The student is expected to:

- (C) use the Pythagorean Theorem and its converse to solve problems
Overview
Students explore the lengths of the sides of a right triangle. Triangles are drawn on grid paper. Horizontal and vertical lengths are determined by counting, and then the length of the hypotenuse is determined.

Students prove the Pythagorean Theorem using three different methods. All of the methods involve drawing squares on each side of the right triangle, subdividing the two smaller squares into triangles or rectangles, cutting out these shapes and placing them on top of the larger square. Students conclude the area of the larger square is equal to the sum of the areas of the two smaller squares. They then will use the Pythagorean Theorem to solve for the length of unknown sides of right triangles set in a variety of contexts.
Warm Up

Solve for $x$.

1. $8^2 + 3^2 = x$
   $x = 73$

2. $36 + x^2 = 85$
   $x = 7$

3. $3^2 + 4^2 = x^2$
   $x = 5$

4. $10^2 + 24^2 = x$
   $x = 676$

5. $48 + x^2 = 95$
   $x \approx 6.9$

6. $6^2 + 8^2 = x^2$
   $x = 10$
Learning Goals
In this lesson, you will:
- Use mathematical properties to discover the Pythagorean Theorem.
- Solve problems involving right triangles.

Key Terms
- right triangle
- right angle
- leg
- hypotenuse
- diagonal of a square
- Pythagorean Theorem
- theorem
- postulate
- proof

What do firefighters and roofers have in common? If you said they both use ladders, you would be correct! Many people who use ladders as part of their job must also take a class in ladder safety. What type of safety tips would you recommend? Do you think the angle of the ladder is important to safety?
Problem 1
Students are given the definitions of a right angle, right triangle, leg and hypotenuse. Using these definitions, they will identify the length of the hypotenuse given three side lengths.

Grouping
Have students complete Questions 1 and 2 independently. Then share the responses as a class.

Share Phase, Questions 1 and 2
• How does the length of the hypotenuse compare to the length of the legs?
• How does the measure of an angle in a right triangle relate to the length of the opposite side?
• How does the length of a side in a right triangle relate to the measure of the opposite angle?
• How does the sum of the lengths of any two sides of a right triangle relate to the length of the third side?

Problem 1  Identifying the Sides of Right Triangles

A right triangle is a triangle with a right angle. A right angle has a measure of 90° and is indicated by a square drawn at the corner formed by the angle. A leg of a right triangle is either of the two shorter sides. Together, the two legs form the right angle of a right triangle. The hypotenuse of a right triangle is the longest side. The hypotenuse is opposite the right angle.

1. The side lengths of right triangles are given. Determine which length represents the hypotenuse.
   a. 5, 12, 13  b. 1, 1, \(\sqrt{2}\)
      The hypotenuse is 13.  The hypotenuse is \(\sqrt{2}\).
   c. 2.4, 5.1, 4.5  d. 75, 21, 72
      The hypotenuse is 5.1.  The hypotenuse is 75.
   e. 15, 39, 36  f. 7, 24, 25
      The hypotenuse is 39.  The hypotenuse is 25.

2. How did you decide which length represented the hypotenuse?
   The hypotenuse is always the longest side of a right triangle.

Can the sides of a right triangle all be the same length?
Problem 2
Students explore the side lengths of right triangles by drawing squares using the sides of the triangle, dividing the two smaller squares into triangles, cutting out the new triangles and placing them on top of the larger square. This proves the area of the larger square is equal to the sum of the areas of the two smaller squares. This activity is repeated but instead of cutting out smaller triangles from the squares, students are instructed to divide the squares into strips, cut out the strips and place the strips on top of the larger square. This also proves the area of the larger square is equal to the sum of the areas of the two smaller squares. A third activity is similar to the first activity but the triangles that determine each square are cut into 4 different congruent triangles. Finally, students write an equation, \( a^2 + b^2 = c^2 \), that represents the relationship among the areas of the squares.

Grouping
Ask a student to read the introduction to Problem 2 aloud. Discuss the definitions and complete the steps to Question 1 as a class.

Problem 2 Exploring Right Triangles
In this problem, you will explore three different right triangles. You will draw squares on each side of the triangles and then answer questions about the completed figures.

A diagonal of a square is a line segment connecting opposite vertices of the square. Let’s explore the side lengths of more right triangles.

1. An isosceles right triangle is drawn on the grid shown on the following page.
   a. A square on the hypotenuse has been drawn for you. Use a straightedge to draw squares on the other two sides of the triangle. Then use different colored pencils to shade each small square.
   b. Draw two diagonals in each of the two smaller squares.
   c. Cut out the two smaller squares along the legs. Then, cut those squares into fourths along the diagonals you drew.
   d. Redraw the squares on the figure in the graphic organizer at the end of Problem 2. Shade the smaller squares again.
   e. Arrange the pieces you cut out to fit inside the larger square on the graphic organizer. Then, tape the triangles on top of the larger square.

Answer these questions in the graphic organizer.

f. What do you notice?

Remember that you can estimate the value of a square root by using the square roots of perfect squares.

\[
\begin{array}{cccccccc}
\sqrt{1} & \sqrt{2} & \sqrt{3} & \sqrt{4} & \sqrt{5} & \sqrt{6} & \sqrt{7} & \sqrt{8} & \sqrt{9} \\
1 & 1.41 & 1.73 & 2 & 2.24 & 2.45 & 2.65 & 2.83 & 3 \\
\end{array}
\]

The square root of 40 is between \( \sqrt{36} \) and \( \sqrt{49} \), or between 6 and 7. \( \sqrt{40} \approx 6.3 \).

Remember, \( A = s^2 \), so, \( \sqrt{A} = s \).

g. Write a sentence that describes the relationship among the areas of the squares.

h. Determine the length of the hypotenuse of the right triangle. Justify your solution.

ELL Tip
Bring students’ attention to the radical sign. Allow students time to share what they understand this mathematical symbol to mean. Reinforce the meaning by reviewing terms such as perfect squares and how this applies to determining the square root of a number. Progress to the estimation of non-perfect square numbers, showing students how the square root of non-perfect squares has a value between two perfect square numbers as modeled in the diagram.
Share Phase, Question 1

- How do the areas of the two smaller squares compare to each other?
- If the areas of the two smaller squares are the same, is the sum of the areas of the 4 triangles in one small square equal to the sum of the areas of the 4 triangles in the other small square? Explain.
- How do the areas of the eight small triangles compare to each other?
- Do all eight triangles fit inside the square drawn along the hypotenuse?
- If all eight triangles fit exactly inside the largest square, what does this imply about the sum of the areas of the eight small triangles and the area of the largest square?
- If all eight triangles fit exactly inside the largest square, what does this imply about the sum of the areas of the two smaller squares and the area of the largest square?
- What is the area of one small square?
- What is sum of the areas of both small squares?
- If the area of each of the small squares is 25 square units, what is the sum of the areas of both small squares?
- If the sum of the areas of both small squares is 50 square units, what is the area of the large square?
- If the area of the large square is 50 square units, what is the length of each side of the square?
Grouping
Have students complete Questions 2 through 5 with a partner. Then share the responses as a class.

Share Phase, Question 2
- Which of the squares is the smallest? Which of the squares is the medium sized square? Which of the squares is the largest?
- How do the areas of the small square and the medium sized square compare to each other?
- What is the area of the smallest square?
- How many 3 unit by 1 unit strips are in the smallest square?
- What is the area of the medium sized square?
- How many 4 unit by 1 unit strips are in the medium square?
- Do all seven strips fit inside the square drawn along the hypotenuse?
- If all seven strips fit exactly inside the largest square, what does this imply about the sum of the areas of the seven strips and the area of the largest square?
- What is sum of the areas of both the small and the medium sized squares?
- If the area of the small square is 9 square units, and the area of the medium sized square is 16 square units, what is the area of the largest square?
- If the area of the large square is 25 square units, what is the length of each side of the square?

2. A right triangle is shown on the following page with one leg 4 units in length and the other leg 3 units in length.
   a. Use a straightedge to draw squares on each side of the triangle. Use different colored pencils to shade each square along the legs.
   b. Cut out the two smaller squares along the legs.
   c. Cut the two squares into strips that are either 4 units by 1 unit or 3 units by 1 unit.
   d. Redraw the squares on the figure in the graphic organizer at the end of Problem 2. Shade the smaller squares again.
   e. Arrange the strips and squares you cut out on top of the square along the hypotenuse on the graphic organizer. You may need to make additional cuts to the strips to create individual squares that are 1 unit by 1 unit. Then, tape the strips on top of the square you drew on the hypotenuse.

Answer these questions in the graphic organizer.
   f. What do you notice?
   g. Write a sentence that describes the relationship among the areas of the squares.
   h. Determine the length of the hypotenuse. Justify your solution.

"Remember, the length of the side of a square is the square root of its area."
The Pythagorean Theorem • 53
2.1 The Pythagorean Theorem

3. A right triangle is shown on the following page with one leg 2 units in length and the other leg 4 units in length.
   a. Use a straightedge to draw squares on each side of the triangle. Use different colored pencils to shade each square along the legs.
   b. Cut out the two smaller squares.
   c. Draw four congruent right triangles on the square with side lengths of 4 units. Then, cut out the four congruent right triangles you drew.
   d. Redraw the squares on the figure in the graphic organizer at the end of Problem 2. Shade the smaller squares again.
   e. Arrange and tape the small square and the 4 congruent triangles you cut out over the square that has one of its sides as the hypotenuse.

Answer these questions in the graphic organizer.

f. What do you notice?

h. Write a sentence that describes the relationship among the areas of the squares.

Determine the length of the hypotenuse. Justify your solution.

4. Compare the sentences you wrote for part (f) in Questions 1, 2, and 3. What do you notice?

I notice that the mathematical sentences are all the same. The area of the larger square is equal to the sum of the areas of the two smaller squares.

5. Write an equation that represents the relationship among the areas of the squares.

Assume that the length of one leg of the right triangle is “a,” the length of the other leg of the right triangle is “b,” and the length of the hypotenuse is “c.”

\[ a^2 + b^2 = c^2 \]

- If the four triangles and smallest square fit exactly inside the largest square, what does this imply about the sum of the areas of the small and medium sized square and the area of the largest square?
- If the area of the small square is 4 square units, and the area of the medium sized square is 16 square units, what is the area of the largest square?
- If the area of the large square is 20 square units, what is the length of each side of the square?
The Pythagorean Theorem

In the diagram, we have a right triangle with sides of lengths 2 units and 4 units. According to the Pythagorean Theorem, the square of the length of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the lengths of the other two sides. Thus, if we denote the lengths of the sides as follows:

- Side 1: 2 units
- Side 2: 4 units
- Hypotenuse: $c$

We can set up the equation:

$$c^2 = 2^2 + 4^2$$

$$c^2 = 4 + 16$$

$$c^2 = 20$$

Therefore, the length of the hypotenuse is $\sqrt{20}$ units.
The Pythagorean Theorem

**Right Triangle: Both Legs with Length of 5 Units**

**What Do You Notice?**
I notice that the eight triangles I cut out and taped fit exactly into the larger square.

**Describe the Relationship Among the Areas of the Squares.**
The area of the larger square is equal to the sum of the areas of the two smaller squares.

The area of the larger square is the sum of the areas of the two smaller squares. That makes the area of the larger square 50 square units. So, the length of the hypotenuse is \( \sqrt{50} \) units, or about 7.1 units.

**Determine the Length of the Hypotenuse.**
Right Triangle: One leg with length of 4 units and the other leg with length of 3 units

What do you notice?
I notice that the strips I cut from the two smaller squares fit exactly on top of the square along the hypotenuse.

Describe the relationship among the areas of the squares.
The area of the larger square is equal to the sum of the areas of the two smaller squares.

The area of the square on the hypotenuse is $16$ square units, or $25$ square units. The hypotenuse is $\sqrt{25}$ units, or exactly 5 units.

Determine the length of the hypotenuse

Pythagorean Theorem
The Pythagorean Theorem

Right Triangle: One leg with length of 2 units and the other leg with length of 4 units

What do you notice?
I notice that the 4 congruent right triangles and the small square I cut out fit exactly on top of the square along the hypotenuse.

Describe the relationship among the areas of the squares.
The area of the larger square is equal to the sum of the areas of the two smaller squares.

What do you notice?
I notice that the 4 congruent right triangles and the small square I cut out fit exactly on top of the square along the hypotenuse.

Determine the length of the hypotenuse.
The area of the square on the hypotenuse is 4 square units + 16 square units, or 20 square units. The hypotenuse is $\sqrt{20}$ units, or about 4.5 units.
Problem 3

After proving this relationship, the Pythagorean Theorem is formally stated. Definitions are given for theorem, postulate, and proof to help students distinguish between a postulate and a theorem.

Grouping

• Ask a student to read the information before Question 1 aloud. Discuss these definitions as a class.
• Have students complete Question 1 with a partner. Then share the responses as a class.

Share Phase, Question 1

• Can the Pythagorean Theorem be used with triangles that are not right triangles?
• If you only know the length of one side of a right triangle, in what situation can the Pythagorean Theorem be used to determine the length of the other two sides?
• Can the Pythagorean Theorem be used if the lengths of two sides of a right triangle are expressed as radicals?
• How do you square a radical?
• What happens when you square a radical?
• When can’t the Pythagorean Theorem be used?

Problem 3  Special Relationships

The special relationship that exists between the squares of the lengths of the sides of a right triangle is known as the Pythagorean Theorem. The sum of the squares of the lengths of the legs of a right triangle equals the square of the length of the hypotenuse.

The Pythagorean Theorem states that if \( a \) and \( b \) are the lengths of the legs of a right triangle and \( c \) is the length of the hypotenuse, then \( a^2 + b^2 = c^2 \).

A theorem is a mathematical statement that can be proven using definitions, postulates, and other theorems. A postulate is a mathematical statement that cannot be proved but is considered true. The Pythagorean Theorem is one of the earliest known to ancient civilization and one of the most famous. This theorem was named after Pythagoras (580 to 496 B.C.), a Greek mathematician and philosopher who was the first to prove the theorem. A proof is a series of steps used to prove the validity of a theorem. While it is called the Pythagorean Theorem, the mathematical knowledge was used by the Babylonians 1000 years before Pythagoras. Many proofs followed that of Pythagoras, including ones proved by Euclid, Socrates, and even the twentieth President of the United States, President James A. Garfield.

1. Use the Pythagorean Theorem to determine the length of the hypotenuse:
   a. in Problem 2, Question 1.
      \[ c^2 = 5^2 + 5^2 \]
      \[ c^2 = 50 \]
      \[ c = \sqrt{50} \]
      The hypotenuse is \( \sqrt{50} \), or about 7.1, units.

   b. in Problem 2, Question 3.
      \[ c^2 = 2^2 + 4^2 \]
      \[ c^2 = 20 \]
      \[ c = \sqrt{20} \]
      The hypotenuse is \( \sqrt{20} \), or about 4.5, units.
Problem 4
Students use the Pythagorean Theorem to solve for the length of unknown sides of right triangles set in a variety of contexts.

Grouping
Have students complete Questions 1 through 6 with a partner. Then share the responses as a class.

Share Phase, Questions 1 and 2
- What is the measure of the angle formed by the building and the ground?
- What kind of triangle is formed by the ladder, the ground and the building?
- Where is the hypotenuse located?
- Where are the legs located?
- Why does the ladder have to be placed at an angle?
- Could you lean the entire ladder up against the building and still climb the ladder?
- In Question 2, what is the measure of the angle formed by the poles and the ground?
- What kind of triangle is formed by the pole, the ground, and the rope?
- Where is the hypotenuse located?
- Where are the legs located?

Problem 4 Maintaining School Grounds

Mitch maintains the Magnolia Middle School campus. Use the Pythagorean Theorem to help Mitch with some of his jobs.

1. Mitch needs to wash the windows on the second floor of a building. He knows the windows are 12 feet above the ground. Because of dense shrubbery, he has to put the base of the ladder 5 feet from the building. What ladder length does he need?

\[5^2 + 12^2 = c^2\]
\[25 + 144 = c^2\]
\[169 = c^2\]
\[13 = c\]

The length of the ladder must be 13 feet.

2. The gym teacher, Ms. Fisher, asked Mitch to put up the badminton net. Ms. Fisher said that the top of the net must be 5 feet above the ground. She knows that Mitch will need to put stakes in the ground for rope supports. She asked that the stakes be placed 6 feet from the base of the poles. Mitch has two pieces of rope, one that is 7 feet long and a second that is 8 feet long. Will these two pieces of rope be enough to secure the badminton poles? Explain your reasoning.

\[5^2 + 6^2 = c^2\]
\[25 + 36 = c^2\]
\[61 = c^2\]
\[7.81 = c\]

The 8-foot rope will work, but the 7-foot rope will be too short. Mitch will have to find another piece of rope to complete the project.
3. Mitch stopped by the baseball field to watch the team practice. The first baseman caught a line drive right on the base. He touched first base for one out and quickly threw the ball to third base to get another out. How far did he throw the ball?

\[ 90^2 + 90^2 = c^2 \]
\[ 8100 + 8100 = c^2 \]
\[ 16,200 = c^2 \]
\[ \sqrt{16,200} = c \]
\[ 127.3 = c \]

The first baseman threw the ball \( \sqrt{16,200} \) feet, or about 127.3 feet.

4. The skate ramp on the playground of a neighboring park is going to be replaced. Mitch needs to determine how long the ramp is to get estimates on the cost of a new skate ramp. He knows the measurements shown in the figure. How long is the existing skate ramp?

\[ 15^2 + 8^2 = c^2 \]
\[ 225 + 64 = c^2 \]
\[ 289 = c^2 \]
\[ 17 = c \]

The skate ramp is 17 feet long.

5. A wheelchair ramp that is constructed to rise 1 foot off the ground must extend 12 feet along the ground. How long will the wheelchair ramp be?

\[ 1^2 + 12^2 = c^2 \]
\[ 1 + 144 = c^2 \]
\[ 145 = c^2 \]
\[ \sqrt{145} = c \]
\[ 12.04 = c \]

The wheelchair ramp will be approximately 12.04 feet long.
Share Phase, Question 6

- In Question 6, how is fencing measured?
- What is a linear foot?
- How can the Pythagorean Theorem be used to solve this problem?

6. The eighth-grade math class keeps a flower garden in the front of the building. The garden is in the shape of a right triangle, and its dimensions are shown. The class wants to install a 3-foot-high picket fence around the garden to keep students from stepping onto the flowers. The picket fence they need costs $5 a linear foot. How much will the fence cost? Do not calculate sales tax. Show your work and justify your solution.

\[ 9^2 + 12^2 = c^2 \]
\[ 81 + 144 = c^2 \]
\[ 225 = c \]
\[ 15 = c \]

The hypotenuse is 15 feet.

\[ 15 + 9 + 12 = 36 \]
They need 36 feet of fencing.

\[ 36 \text{ ft} \times \$5/\text{ft} = \$180 \]
The picket fence will cost $180 before sales tax.
Problem 5
Students use the Pythagorean Theorem to create an equation and solve the equation for the length of unknown measurements.

Grouping
Have students complete Question 1 with a partner. Then share the responses as a class.

Share Phase, Question 1
• Are you solving for the length of a leg or the hypotenuse?
• How is solving for the length of a hypotenuse different than solving for the length of a leg?
• Is it easier to solve for the length of a leg or the length of the hypotenuse? Explain.
• How can the Pythagorean Theorem be rewritten so it is easier to solve for the length of a leg?
• If $a^2 + b^2 = c^2$, does $b^2 + c^2 = a^2$? Explain.
• If $a^2 + b^2 = c^2$, does $a^2 + c^2 = b^2$? Explain.

Be prepared to share your solutions and methods.
Follow Up

Assignment
Use the Assignment for Lesson 2.1 in the Student Assignments book. See the Teacher’s Resources and Assessments book for answers.

Skills Practice
Refer to the Skills Practice worksheet for Lesson 2.1 in the Student Assignments book for additional resources. See the Teacher’s Resources and Assessments book for answers.

Assessment
See the Assessments provided in the Teacher’s Resources and Assessments book for Chapter 2.

Check for Students’ Understanding
A rectangular swimming pool is 24 meters by 10 meters.

1. Draw the pool and include the dimensions.

2. Describe the angles formed at each corner of the pool.
The corner angles are right angles. Each angle has the measure of 90°.

3. Jane said she could swim from corner to corner without taking a breath. Carli said she could swim much further than Jamie and still swim from corner to corner. Determine the distances Jane and Carli swam.

\[ a^2 + b^2 = c^2 \]
\[ 10^2 + 24^2 = c^2 \]
\[ 100 + 576 = c^2 \]
\[ 676 = c^2 \]
\[ 26 = c \]

Carli swam the length of the diagonal of the rectangular pool: 26 meters.

Jane swam either the length or the width of the pool: 10 or 24 meters.
Learning Goals
In this lesson, you will:

- Use the Pythagorean Theorem and the Converse of the Pythagorean Theorem to determine unknown side lengths in right triangles.

Key Terms
- converse
- Converse of the Pythagorean Theorem
- Pythagorean triple

Essential Ideas
- The converse of a theorem is created when the if-then parts of that theorem are exchanged.
- The Converse of the Pythagorean Theorem states; if \( a^2 + b^2 = c^2 \), then the triangle is a right triangle.
- A Pythagorean triple is any set of three positive integers that satisfy the equation \( a^2 + b^2 = c^2 \).

Texas Essential Knowledge and Skills for Mathematics
Grade 8
(7) Expressions, equations, and relationships. The student applies mathematical process standards to use geometry to solve problems. The student is expected to:

(C) use the Pythagorean Theorem and its converse to solve problems

(D) determine the distance between two points on a coordinate plane using the Pythagorean Theorem
Overview
Students are introduced to the terms converse, Pythagorean Triple, and the Converse of the Pythagorean Theorem. They use the Pythagorean Theorem to determine if three side lengths form a right triangle. If the Pythagorean Theorem is satisfied, the triangle must be a right triangle. Students are given a definition of Pythagorean triple, and complete a table composed of multiples of the Pythagorean triple, 3, 4, 5. They then generate their own Pythagorean triple, and complete a table containing the multiples of that triple. Finally, students will use either the Pythagorean Theorem or the Converse of the Pythagorean Theorem to solve problems.
Warm Up

A bird leaves its nest and flies 3 miles due south, 2 miles due east, 5 miles due south and 1 mile due east to visit a friend’s nest.

1. Draw a model of the situation.

2. Determine the distance between both nests.

\[
8^2 + 3^2 = c^2 \\
64 + 9 = c^2 \\
73 = c^2 \\
c = \sqrt{73} \approx 8.5
\]
Mind your \( p \)'s and \( q \)'s!" This statement usually refers to reminding a person to watch their manners. While the definition is easy to understand, the origin of this saying is not clear. Some people think that it comes from a similar reminder for people to remember their “please and thank-yous” where the “\( q \)'s” rhymes with “yous.” Others believe that it was a reminder to young children not to mix up \( p \)'s and \( q \)'s when writing because both letters look very similar.

However, maybe the origin of this saying comes from math. When working with theorems (as you did in the last lesson), mathematicians encounter if-then statements. Often, if-then statements are defined as “if \( p \), then \( q \),” with the \( p \) representing an assumption and the \( q \) representing the outcome of the assumption. So, just maybe math played a role in this saying.
Problem 1
The Converse of the Pythagorean Theorem is provided, and students use this theorem to determine whether the triangles with given side lengths result in a right triangle. Pythagorean triples are introduced and students will compute multiples of the Pythagorean triple 3-4-5, and generate a different Pythagorean triple.

**Problem 1** The Converse

The Pythagorean Theorem can be used to solve many problems involving right triangles, squares, and rectangles. The Pythagorean Theorem states that in a right triangle, the square of the hypotenuse length equals the sum of the squares of the leg lengths. In other words, if you have a right triangle with a hypotenuse of length \( c \) and legs of lengths \( a \) and \( b \), then \( a^2 + b^2 = c^2 \).

The converse of a theorem is created when the if-then parts of that theorem are exchanged.

The Converse of the Pythagorean Theorem states that if \( a^2 + b^2 = c^2 \), then the triangle is a right triangle.

If the lengths of the sides of a triangle satisfy the equation \( a^2 + b^2 = c^2 \), then the triangle is a right triangle.

1. Determine whether the triangle with the given side lengths is a right triangle.
   a. 9, 12, 15
      \[ 9^2 + 12^2 = 15^2 \]
      \[ 81 + 144 = 225 \]
      \[ 225 = 225 \]
      This is a right triangle.
   b. 24, 45, 51
      \[ 24^2 + 45^2 = 51^2 \]
      \[ 576 + 2025 = 2601 \]
      \[ 2601 = 2601 \]
      This is a right triangle.
   c. 25, 16, 9
      \[ 9^2 + 16^2 = 25^2 \]
      \[ 81 + 256 = 625 \]
      \[ 337 = 625 \]
      This is not a right triangle because 337 is not equal to 625.
   d. 8, 8, 11
      \[ 8^2 + 8^2 = 11^2 \]
      \[ 64 + 64 = 121 \]
      \[ 128 \neq 121 \]
      The triangle is not a right triangle because 128 is not equal to 121.

**Grouping**
- Ask a student to read the introduction to Problem 1 aloud. Discuss the definitions as a class.
- Have students complete Question 1 with a partner. Then share the responses as a class.

**Discuss Phase, Introduction**
- What is an example of an if-then statement and an example of the converse of that statement?
- What is the converse of the statement “If today is Saturday or Sunday, then I do not have to go to school today.”
- Can an if-then statement be true, if the converse of that statement is false? Explain.
- Can an if-then statement be false, if the converse of that statement is true? Explain.
Share Phase, Question 1

- Which side length most likely represents the hypotenuse? Why?
- Is the sum of 81 and 144 equal to 225? What does this tell you?
- Is the sum of 576 and 2050 equal to 2601? What does this tell you?
- Is the sum of 81 and 256 equal to 625? What does this tell you?
- Is the sum of 64 and 64 equal to 121? What does this tell you?

Grouping

- Ask a student to read the information prior to Question 2 aloud. Discuss the definition as a class.
- Have students complete Questions 2 through 4 with a partner. Then share the responses as a class.

Share Phase, Questions 2 through 4

- Is 7-24-25 a Pythagorean triple? Explain.
- Can any two integers be used to form a Pythagorean triple? Why or why not?
- Can 2 and 3 be used to form a Pythagorean triple? Why or why not?
- How many Pythagorean triples are there?
Problem 2
Students apply the Pythagorean Theorem and the Converse of the Pythagorean Theorem to solve problems.

Grouping
Have students complete Questions 1 through 12 with a partner. Then share the responses as a class.

Share Phase, Questions 1 through 3
- What is a brace? Why do carpenters use braces?
- A diagonal brace on a rectangular structure divides the rectangle into what shapes?
- Can you use the Pythagorean Theorem or the Converse of the Pythagorean Theorem to solve this problem? Which one do you need? How do you know it is a right triangle?
- Are you solving for the length of a leg or the length of the hypotenuse?
- In Question 2, if the length of a diagonal squared of a quadrilateral does not equal the sum of the squared lengths of the length and width, what does this tell you about the quadrilateral?
- Do you need to use the Pythagorean Theorem or the Converse of the Pythagorean Theorem to solve this problem?

1. A carpenter attaches a brace to a rectangular-shaped picture frame. If the dimensions of the picture frame are 30 inches by 40 inches, what is the length of the brace?

The length of the brace is the diagonal of the rectangle, which is 50 inches.

2. Bill is building a rectangular deck that will be 8 feet wide and 15 feet long. Tyrone is helping Bill with the deck. Tyrone has two boards, one that is 8 feet long and one that is 7 feet long. He puts the two boards together, end to end, and lays them on the diagonal of the deck area, where they just fit. What should he tell Bill?

The diagonal of the rectangle needs to be 17 feet. Because the two boards’ combined lengths are 15 feet and not 17 feet, Tyrone should tell Bill that the deck is not a rectangle. The corners of the deck are not right angles.

3. A television is identified by the diagonal measurement of the screen. A television has a 36-inch screen whose height is 22 inches. What is the length of the television screen? Round your answer to the nearest inch.

The length of the television screen, rounded to the nearest inch, is 28 inches.

- In Question 3, are the corner angles of a television screen always right angles?
- Is the length of the diagonal of a rectangular television screen always longer than the width of the screen?
- Why are televisions referred to by the length of their diagonal?
- Are you solving for the length of a leg or the length of a hypotenuse?
Share Phase, Questions 4 through 6

• Is the circular glass table top taller than the doorway?
• Do you need to know the height of the base of the table to determine if it will fit through the doorway?
• Do you need to use the Pythagorean Theorem or the Converse of the Pythagorean Theorem to solve this problem?
• In Question 5, what are stretcher bars?
• If the painting is rectangular, does this guarantee the corners of the painting are right angles?
• If the rectangular painting is stretched, could the measures of the angles formed in the corners change?
• Do you need to use the Pythagorean Theorem or the Converse of the Pythagorean Theorem to solve this problem?
• Are you solving for the length of a leg or the length of a hypotenuse?
• In Question 6, what kind of triangle is formed by the ladder, the ground, and the building?
• Where is the hypotenuse in the situation?
• Where are the legs in the situation?
• Could a longer ladder be used if it was placed further away from the edge of a building?

4. Orville and Jerri want to put a custom-made, round table in their dining room. The table top is made of glass with a diameter of 85 inches. The front door is 36 inches wide and 80 inches tall. Orville thinks the table top will fit through the door, but Jerri does not. Who is correct and why?

Orville is correct. The table top can be taken through the door on the diagonal. The door frame is a rectangle with dimensions of 36 inches by 80 inches. Therefore, the diagonal of the doorway is approximately 87.7 inches. There will be approximately 2.7 inches to spare.

5. Sherie makes a canvas frame for a painting using stretcher bars. The rectangular painting will be 12 inches long and 9 inches wide. How can she use a ruler to make sure that the corners of the frame will be right angles?

Sherie can measure the diagonal. The diagonal must be 15 inches long, or the angles are not right angles.

6. A 10-foot ladder is placed 4 feet from the edge of a building. How far up the building does the ladder reach? Round your answer to the nearest tenth of a foot.

\[
4^2 + b^2 = 10^2 \\
16 + b^2 = 100 \\
b^2 = 84 \\
b = \sqrt{84} \\
b \approx 9.2
\]

The ladder reaches about 9.2 feet up the edge of the building.
Share Phase, Questions 7 and 8

- In Question 7, what does slant length mean?
- Where are support poles of a tent usually located?
- How is the height of the support pole of a tent determined?
- Do you need to use the Pythagorean Theorem or the Converse of the Pythagorean Theorem to solve this problem?
- Are you solving for the length of a leg or the length of a hypotenuse?
- In Question 8, how do kilometers compare to miles?
- What is the difference between north and due north?
- Do you need to use the Pythagorean Theorem or the Converse of the Pythagorean Theorem to solve this problem?
- Are you solving for the length of a leg or the length of a hypotenuse?

7. Chris has a tent that is 64 inches wide with a slant length of 68 inches on each side. What is the height of the center pole needed to prop up the tent?

Because the pole must be in the center, a right triangle can be formed with a leg of 32 inches and a hypotenuse of 68 inches. The height of the center pole needs to be 60 inches.

8. A ship left shore and sailed 240 kilometers east, turned due north, then sailed another 70 kilometers. How many kilometers is the ship from shore by the most direct path?

\[240^2 + 70^2 = c^2\]
\[57,600 + 4900 = c^2\]
\[62,500 = c^2\]
\[\sqrt{62,500} = c\]
\[250 = c\]

The ship is 250 kilometers from its departure point on shore.
Share Phase, Questions 9 through 11

- In Question 9, what does directly south mean?
- Do you need to use the Pythagorean Theorem or the Converse of the Pythagorean Theorem to solve this problem?
- Are you solving for the length of a leg or the length of a hypotenuse?
- In Question 10, what is the difference between east and due east?
- Do you need to use the Pythagorean Theorem or the Converse of the Pythagorean Theorem to solve this problem?
- In Question 11, do you need to use the Pythagorean Theorem or the Converse of the Pythagorean Theorem to solve this problem?

9. Tonya walks to school every day. She must travel 4 blocks east and 3 blocks south around a parking lot. Upon arriving at school, she realizes that she forgot her math homework. In a panic, she decides to run back home to get her homework by taking a shortcut through the parking lot.

![Diagram of Tonya's House, Parking Lot, and School]

a. Describe how many blocks long Tonya's shortcut is.
   Tonya's shortcut is 5 blocks.

b. How many fewer blocks did Tonya walk by taking the shortcut?
   Tonya walked 2 fewer blocks than her usual route. $7 - 5 = 2$

10. Danielle walks 88 feet due east to the library from her house. From the library, she walks 187 feet northwest to the corner store. Finally, she walks 57 feet from the corner store back home. Does she live directly south of the corner store? Justify your answer.

   To solve this problem, I must determine whether $88^2 + 57^2 = 187^2$.
   
   $7744 + 3249 = 10,993$
   $187^2 = 34,969$
   
   Because $187^2$ does not equal 10,993, Danielle does not live directly south of the corner store.

11. What is the diagonal length of a square that has a side length of 10 cm?

   $10^2 + 10^2 = c^2$
   $100 + 100 = c^2$
   $200 = c^2$
   $c = \sqrt{200}$
   $c = 14.14$

   The diagonal length of a square with a side length of 10 cm is about 14.14 cm.
Share Phase, Question 12

• In Question 12, do you need to use the Pythagorean Theorem or the Converse of the Pythagorean Theorem to solve this problem?

• Where is the right triangle needed to solve this problem?

• Which answer is more exact: the answer that is expressed using a radical or the answer that is expressed using a decimal?

• In Question 13, do you need to use the Pythagorean Theorem or the Converse of the Pythagorean Theorem to solve this problem?

• If you were not given the lengths of any sides of the right triangle, how can you use the Pythagorean Theorem or the Converse of the Pythagorean Theorem?

• How is the perimeter of the triangle determined?

12. Calculate the length of the segment that connects the points (1, -5) and (3, 6).

a. Write your answer as a radical.

\[ 2^2 + 11^2 = c^2 \]
\[ 4 + 121 = c^2 \]
\[ 125 = c^2 \]
\[ \sqrt{125} = c \]

The length of the segment is \(\sqrt{125}\).

b. Write your answer as a decimal rounded to the nearest hundredth.

The length of the segment rounded to the nearest hundredth is 11.18.

Be prepared to share your solutions and methods.
Follow Up

Assignment
Use the Assignment for Lesson 2.2 in the Student Assignments book. See the Teacher’s Resources and Assessments book for answers.

Skills Practice
Refer to the Skills Practice worksheet for Lesson 2.2 in the Student Assignments book for additional resources. See the Teacher’s Resources and Assessments book for answers.

Assessment
See the Assessments provided in the Teacher’s Resources and Assessments book for Chapter 2.

Check for Students’ Understanding
Create a Pythagorean triple that contains each length.

1. 3
   3-4-5

2. 12
   5-12-13

3. 15
   8-15-17

4. 24
   7-24-25

5. 20
   20-21-29

6. 40
   9-40-41

7. Can any integer be used to create a Pythagorean triple? Explain.
   No. Not every integer can be used to create a Pythagorean triple. For example, 2 cannot be used to create a Pythagorean triple.
Learning Goals
In this lesson, you will:

➢ Use the Pythagorean Theorem and the Converse of the Pythagorean Theorem to determine the unknown side lengths in right triangles.

Essential Ideas

➢ The Pythagorean Theorem and the Converse of the Pythagorean Theorem are used to determine unknown lengths.

➢ The Pythagorean Theorem states; if \(a\) and \(b\) are the lengths of the legs of a right triangle and \(c\) is the length of the hypotenuse, then \(a^2 + b^2 = c^2\).

➢ The Converse of the Pythagorean Theorem states; if \(a^2 + b^2 = c^2\), then the triangle is a right triangle.

Texas Essential Knowledge and Skills for Mathematics

Grade 8

(7) Expressions, equations, and relationships. The student applies mathematical process standards to use geometry to solve problems. The student is expected to:

(C) use the Pythagorean Theorem and its converse to solve problems

2.3 Solving for Unknown Lengths
Overview
Students apply the Pythagorean Theorem and the Converse of the Pythagorean Theorem to solve problems. All answers are rounded to the nearest tenth.
Warm Up

A plane is 5 miles directly above a house and 42 miles from the nearest airport.

1. Draw a model of the situation using a right triangle.

   ![Diagram of a right triangle with a plane 5 miles above a house and 42 miles from the nearest airport.]

2. Where is the right angle in this situation?
   The right angle is formed by the segments representing the distance from the house to the airport and the distance from the house to the plane.

3. Determine the distance from the house to the airport.
   
   $5^2 + b^2 = 42^2$
   
   $25 + b^2 = 1764$
   
   $1739 = b^2$
   
   $c = \sqrt{1739} \approx 41.7$
   
   The house is about 41.7 miles from the airport.

4. Did you use the Pythagorean Theorem or the Converse of the Pythagorean Theorem to solve this problem? Explain your reasoning.
   Since I was solving for the length of an unknown side of a right triangle, I used the Pythagorean Theorem to solve this problem.
Learning Goal

In this lesson, you will:

- Use the Pythagorean Theorem and the Converse of the Pythagorean Theorem to determine the unknown side lengths in right triangles.

There's a very famous mathematical scene in the movie *The Wizard of Oz*. At the end, when the wizard helps the scarecrow realize that he has had a brain all along, the scarecrow says this:

“The sum of the square roots of any two sides of an isosceles triangle is equal to the square root of the remaining side. Oh joy! Rapture! I've got a brain! How can I ever thank you enough?”

What did the scarecrow get wrong?
Problem 1
Students use the Pythagorean Theorem to determine the length of the hypotenuse of a right triangle, given the lengths of two legs. Answers are rounded to the nearest tenth.

Grouping
Have students complete Questions 1 through 4 with a partner. Then share the responses as a class.

Share Phase, Questions 1 through 4
• Were any values of \( c^2 \) a perfect square? Which ones?
• Were any values of \( c^2 \) not a perfect square? Which ones?
• How did you calculate the value of \( c \) when \( c^2 \) was not a perfect square?
• \( \sqrt{128} \) is between what two perfect squares?
• \( \sqrt{325} \) is between what two perfect squares?
**Problem 2**

Students use the Pythagorean Theorem to determine the length of a leg of a right triangle, given the lengths of one leg and the hypotenuse. Answers are rounded to the nearest tenth.

**Grouping**

Have students complete Questions 1 through 4 with a partner. Then share the responses as a class.

**Share Phase, Questions 1 through 4**

- Were any values of \(a^2\) or \(b^2\) a perfect square? Which ones?
- Were any values of \(a^2\) or \(b^2\) not a perfect square? Which ones?
- How did you calculate the value of \(a^2\) or \(b^2\) when \(a^2\) or \(b^2\) was not a perfect square?
- \(\sqrt{75}\) is between what two perfect squares?
- \(\sqrt{72}\) is between what two perfect squares?

---

**Problem 2  Determining the Length of a Leg**

Determine the unknown leg length. Round your answer to the nearest tenth, if necessary.

1. \(a = 5\)  \(b = 13\)
   \[5^2 + b^2 = 13^2\]
   \[25 + b^2 = 169\]
   \[b^2 = 144\]
   \[b = 12\]
   The unknown leg length is 12 units.

2. \(a = 15\)
   \[a^2 + 12^2 = 15^2\]
   \[a^2 + 144 = 225\]
   \[a^2 = 81\]
   \[a = 9\]
   The unknown leg length is 9 units.

3. \(a = 10\)
   \[a^2 + 5^2 = 10^2\]
   \[a^2 + 25 = 100\]
   \[a^2 = 75\]
   \[a = \sqrt{75}\]
   \[a = 8.7\]
   The unknown leg length is approximately 8.7 units.

4. \(a = 3\)  \(b = 9\)
   \[3^2 + b^2 = 9^2\]
   \[9 + b^2 = 81\]
   \[b^2 = 72\]
   \[b = \sqrt{72}\]
   \[b = 8.5\]
   The unknown leg length is approximately 8.5 units.
Problem 3
Students use the Converse of the Pythagorean Theorem to determine if the triangle is a right triangle, given the lengths of three sides. Answers are rounded to the nearest tenth.

Problem 3  Determining the Right Triangle

Use the converse of the Pythagorean Theorem to determine whether each triangle is a right triangle. Explain your answer.

1. 

Yes. This is a right triangle.
\[8^2 + 15^2 = 64 + 225 = 289 = 17^2\]
The sum of the squares of the lengths of the two legs is equal to the square of the length of the hypotenuse, so this is a right triangle.

2. 

No. This is not a right triangle.
\[5^2 + 7^2 = 25 + 49 = 74\]
\[9^2 = 81\]
\[74 \neq 81\]
The sum of the squares of the lengths of the two legs is not equal to the square of the length of the hypotenuse, so this is not a right triangle.

3. 

No. This is not a right triangle.
\[4^2 + 8^2 = 16 + 64 = 80\]
\[10^2 = 100\]
\[80 \neq 100\]
The sum of the squares of the lengths of the two legs is not equal to the square of the length of the hypotenuse, so this is not a right triangle.

4. 

Yes. This is a right triangle.
\[30^2 + 40^2 = 900 + 1600 = 2500 = 50^2\]
The sum of the squares of the lengths of the two legs is equal to the square of the length of the hypotenuse, so this is a right triangle.

Assign students one the four problems in Problem 3. Explain to students that they will be using the converse of the Pythagorean Theorem to determine if the triangle is a right triangle. Ask students to demonstrate their understanding by using the mathematical theorem, but instead of explaining aloud, allow Beginning English Language Learners to write “yes,” or “no” to show whether or not the triangle is a right triangle.

Grouping
Have students complete Questions 1 through 4 with a partner. Then share the responses as a class.

Share Phase, Questions 1 through 4
- Which value represents the hypotenuse?
- How did you set up the equation?
- How do you know when the triangle is or is not a right triangle?
- How does the Converse of the Pythagorean Theorem help to solve this problem?
Problem 4

Students apply the Pythagorean Theorem in four situations to determine an unknown measurement. Answers are rounded to the nearest tenth.

Grouping

Have students complete Questions 1 through 4 with a partner. Then share the responses as a class.

Share Phase, Questions 1 through 4

- Do you need a diagram to help you solve this problem?
- What information in the problem helps you determine what represents the hypotenuse?
- What does the hypotenuse represent in the problem situation?
- Did you need a calculator to determine the answer? Why or why not?
- Which answers are the results of a perfect square?
- When the answer was not the result of a perfect square, what two perfect squares does the answer lie between?
- How do you know you used the Pythagorean Theorem correctly?
- If the Pythagorean Theorem was used incorrectly, what was the most likely error?

Problem 4 Determining the Unknown Length

Use the Pythagorean Theorem to calculate each unknown length. Round your answer to the nearest tenth, if necessary.

1. Chandra has a ladder that is 20 feet long. If the top of the ladder reaches 16 feet up the side of a building, how far from the building is the base of the ladder?

$$16^2 + b^2 = 20^2$$
$$256 + b^2 = 400$$
$$b^2 = 400 - 256$$
$$b^2 = 144$$
$$b = \sqrt{144}$$
$$b = 12$$

The base of the ladder is 12 feet from the building.

2. A scaffold has a diagonal support beam to strengthen it. If the scaffold is 12 feet high and 5 feet wide, how long must the support beam be?

$$5^2 + 12^2 = c^2$$
$$25 + 144 = c^2$$
$$169 = c^2$$
$$\sqrt{169} = c$$
$$13 = c$$

The length of the diagonal support beam must be 13 feet.
3. The length of the hypotenuse of a right triangle is 40 centimeters. The legs of the triangle are the same length. How long is each leg of the triangle?

\[ a^2 + a^2 = 40^2 \]
\[ 2a^2 = 1600 \]
\[ a^2 = 800 \]
\[ a = \sqrt{800} \]
\[ a = 28.3 \]

The length of each leg of the triangle is approximately 28.3 centimeters.

4. A carpenter props a ladder against the wall of a building. The base of the ladder is 10 feet from the wall. The top of the ladder is 24 feet from the ground. How long is the ladder?

\[ 10^2 + 24^2 = c^2 \]
\[ 100 + 576 = c^2 \]
\[ 676 = c^2 \]
\[ \sqrt{676} = c \]
\[ 26 = c \]

The ladder is 26 feet long.

Be prepared to share your solutions and methods.
Follow Up

Assignment
Use the Assignment for Lesson 2.3 in the Student Assignments book. See the Teacher's Resources and Assessments book for answers.

Skills Practice
Refer to the Skills Practice worksheet for Lesson 2.3 in the Student Assignments book for additional resources. See the Teacher's Resources and Assessments book for answers.

Assessment
See the Assessments provided in the Teacher's Resources and Assessments book for Chapter 2.

Check for Students' Understanding
A boat drops an anchor with a 200-foot chain. The lake is 75 feet deep.

1. Draw a model of the situation using a right triangle.

![](image)

2. Where is the right angle in this situation?
   
   The right angle is formed by the segments representing the distance from the boat to the bottom of the lake and the distance the boat will drift on the water surface.

3. Determine the farthest distance the boat can drift on the surface in any direction.
   
   \[75^2 + b^2 = 200^2\]
   \[5625 + b^2 = 40000\]
   \[34375 = b^2\]
   \[c = \sqrt{34375} \approx 185.4\]
   
   The boat can drift on the surface of the water about 185.4 feet in any direction.

4. Did you use the Pythagorean Theorem or the Converse of the Pythagorean Theorem to solve this problem? Explain your reasoning.
   
   Since I was solving for the length of an unknown side of a right triangle, I used the Pythagorean Theorem to solve this problem.
Meeting Friends
The Distance Between Two Points in a Coordinate System

Learning Goals
In this lesson, you will:
▶ Use the Pythagorean Theorem to determine the distance between two points in a coordinate system.

Essential Ideas
• The Pythagorean Theorem is used to determine the distance between two points in a coordinate plane.
• The Pythagorean Theorem states; if $a$ and $b$ are the lengths of the legs of a right triangle and $c$ is the length of the hypotenuse, then $a^2 + b^2 = c^2$.

Texas Essential Knowledge and Skills for Mathematics
Grade 8
(7) Expressions, equations, and relationships. The student applies mathematical process standards to use geometry to solve problems. The student is expected to:

(D) determine the distance between two points on a coordinate plane using the Pythagorean Theorem.
Overview
Students apply the Pythagorean Theorem to a situation on a coordinate plane. They calculate various distances using coordinates of points aligned either horizontally or vertically using subtraction and diagonal distances using the Pythagorean Theorem. Students are then given four problems in which two points are plotted on a coordinate plane and they will use the Pythagorean Theorem to determine the distance between the two given points.
Warm Up

Use the coordinate plane shown to answer each question.

1. Plot points A (−6, −2) and B (−6, −8).

2. Is the distance between points A and B considered a horizontal distance, a vertical distance, or a diagonal distance? Explain your reasoning.
   The distance between points A and B is a vertical distance because the points are directly above or below each other.

3. How do you calculate the distance between points A and B?
   To calculate the distance between points A and B, subtract 2 from 8.

4. What is the distance between points A and B?
   The distance between points A and B is equal to 6 units.

5. How do the negative coordinates affect the distance between points A and B?
   The negative coordinates have no affect on the distance between the two points. Distance is always a positive value.
Learning Goal
In this lesson, you will:
- Use the Pythagorean Theorem to determine the distance between two points in a coordinate system.

Try this in your class. All you need is a regulation size basketball (29.5 inches in diameter), a tennis ball, and a tape measure. Have your teacher hold the basketball, and give the tennis ball to a student. The basketball represents the Earth, and the tennis ball represents the Moon.

Here’s the question each student should guess at: How far away from the basketball should you hold the tennis ball so that the distance between the two represents the actual distance between the Earth and the Moon to scale?

Use the tape measure to record each student’s guess. Have your teacher show you the answer when you’re done. See who can get the closest.
Problem 1
The locations of friends’ houses and a bookstore are described using street names in a neighborhood laid out on a square grid. Students determine the location of each on the coordinate plane and calculate distances between locations using either subtraction (for vertical or horizontal distances) or the Pythagorean Theorem (for diagonal distances).

Grouping
• Ask a student to read the introduction to Problem 1 aloud. Discuss the grid as a class.
• Have students complete Questions 1 through 4 with a partner. Then share the responses as a class.

Share Phase, Questions 1 through 4
• How did you determine how far Shawn walks to get to the bookstore?
• Does Shawn walk to the bookstore in a vertical, horizontal, or diagonal direction?
• How did you determine how far Tamara walks to get to the bookstore?
• Does Tamara walk to the bookstore in a vertical, horizontal, or diagonal direction?

Problem 1 Meeting at the Bookstore
Two friends, Shawn and Tamara, live in a city in which the streets are laid out in a grid system.
Shawn lives on Descartes Avenue and Tamara lives on Elm Street as shown. The two friends often meet at the bookstore. Each grid square represents one city block.

1. How many blocks does Shawn walk to get to the bookstore?
   Shawn walks 4 blocks to get to the bookstore.

2. How many blocks does Tamara walk to get to the bookstore?
   Tamara walks 5 blocks to get to the bookstore.

3. Determine the distance, in blocks, Tamara would walk if she traveled from her house to the bookstore and then to Shawn’s house.
   \[5 + 4 = 9\]
   Tamara would walk 9 blocks.

4. Determine the distance, in blocks, Tamara would walk if she traveled in a straight line from her house to Shawn’s house. Explain your calculation. Round your answer to the nearest tenth of a block.
   Let \(d\) be the distance between Tamara’s house and Shawn’s house.
   \[d^2 = 5^2 + 4^2\]
   \[d^2 = 25 + 16\]
   \[d^2 = 41\]
   \[d = 6.4\]
   Tamara would walk about 6.4 blocks.
   I used the Pythagorean Theorem to calculate the distance between Tamara’s house and Shawn’s house.

• What operation was used to determine how far Tamara walked when she walked to Shawn’s house by way of the bookstore?
• Does Tamara walk to her house from Shawn’s house in a vertical, horizontal, or diagonal direction?
• What distance in this situation is represented by the hypotenuse?
• What two perfect squares does this distance lie between?
Grouping
Have students complete Questions 5 through 14 with a partner. Then share the responses as a class.

Share Phase, Questions 5 through 7
• What are the coordinates of Don’s house?
• What are the coordinates of Freda’s house?
• What are the coordinates of Bert’s house?
• Whose houses are horizontally aligned?
• Whose houses are vertically aligned?
• Whose houses are diagonally aligned?

5. Don, a friend of Shawn and Tamara, lives three blocks east of Descartes Avenue and five blocks north of Elm Street. Freda, another friend, lives seven blocks east of Descartes Avenue and two blocks north of Elm Street. Plot the location of Don’s house and Freda’s house on the grid. Label each location and label the coordinates of each location.

a. Name the streets that Don lives on.
Don lives at the intersection of Euclid Avenue and Oak Street.

b. Name the streets that Freda lives on.
Freda lives at the intersection of Euler Avenue and Maple Street.

6. Another friend, Bert, lives at the intersection of the avenue that Don lives on and the street that Freda lives on. Plot the location of Bert’s house on the grid in Question 5 and label the coordinates. Describe the location of Bert’s house with respect to Descartes Avenue and Elm Street.
Bert lives 3 blocks east of Descartes Avenue and 2 blocks north of Elm Street.

7. How do the coordinates of Bert’s house compare to the coordinates of Don’s house and Freda’s house?
The first coordinate of Bert’s house is the same as the first coordinate of Don’s house. The second coordinate of Bert’s house is the same as the second coordinate of Freda’s house.
Share Phase, Questions 8 through 14

- What expression is used to represent the distance between Don’s house and Bert’s house?
- What expression is used to represent the distance between Bert’s house and Freda’s house?
- What expression is used to represent the distance between Freda’s house and Don’s house and the distance from Bert’s house to Don’s house?
- If Freda walks to Don’s house on this path, is she walking a horizontal, vertical, or diagonal distance?
- Do the distances between Freda’s house, Bert’s house, and Don’s house form a right triangle? How do you know?
- The distance between which two houses is represented by the hypotenuse?

8. Use the house coordinates to write and evaluate an expression that represents the distance between Don’s and Bert’s houses.
   \[ 5 - 2 = 3 \]

9. How far, in blocks, does Don have to walk to get to Bert’s house?
   
   Don has to walk 3 blocks to get to Bert’s house.

10. Use the house coordinates to write an expression that represents the distance between Bert’s and Freda’s houses.
    
    \[ 7 - 3 = 4 \]

11. How far, in blocks, does Bert have to walk to get to Freda’s house?
    
    Bert has to walk 4 blocks to get to Freda’s house.

12. All three friends meet at Don’s house to study geometry. Freda walks to Bert’s house, and then they walk together to Don’s house. Use the coordinates to write and evaluate an expression that represents the distance from Freda’s house to Bert’s house and from Bert’s house to Don’s house.
    
    \[ (7 - 3) + (5 - 2) = 4 + 3 = 7 \]

13. How far, in blocks, does Freda walk altogether?
    
    Freda walks 7 blocks.

14. Draw the direct path from Don’s house to Freda’s house on the coordinate plane in Question 5. If Freda walks to Don’s house on this path, how far, in blocks, does she walk? Explain how you determined your answer.

   Let \( d \) be the direct distance between Don’s house and Freda’s house.
   
   \[ d^2 = 3^2 + 4^2 \]
   
   \[ d^2 = 9 + 16 \]
   
   \[ d^2 = 25 \]
   
   \[ d = 5 \]

   Freda walks 5 blocks.

I used the Pythagorean Theorem.
Problem 2
Two points that are not aligned horizontally or vertically are given on a coordinate plane. Students connect the points and treat the segment as a hypotenuse by locating a third point to form a right triangle. After all points are connected, students will use the Pythagorean Theorem to determine the length of the hypotenuse or the distance between the two given points.

Note
It is important that students recognize there are two possible locations for the third point. The coordinates of the third point could be (3, 2) or (1, 7).

Grouping
Have students complete Question 1 with a partner. Then share the responses as a class.

Share Phase, Question 1
- Can the third point of the triangle be located in more than one place? Explain.
- Does the alternate location of the third point change the situation? Explain.
- Will the alternate location of the third point change the lengths of the legs of the right triangle? Explain.
- Will the alternate location of the third point change the length of the hypotenuse of the right triangle? Explain.

Is the value of $c^2$ a perfect square?
- If the value of $c^2$ is not a perfect square, the value of $c$ lies between which two perfect squares?
Grouping
Have students complete Questions 2 through 5 with a partner. Then share the responses as a class.

Share Phase, Question 2
- What is an alternate location for the third point?
- Is the value of $c^2$ a perfect square?
- If the value of $c^2$ is not a perfect square, the value of $c$ lies between which two perfect squares?

Determine the distance between each pair of points by graphing and connecting the points, creating a right triangle, and applying the Pythagorean Theorem.

2. (3, 4) and (6, 8)

\[ a^2 + b^2 = c^2 \]
\[ 3^2 + 4^2 = c^2 \]
\[ 9 + 16 = c^2 \]
\[ c^2 = 25 \]
\[ c = 5 \]

The distance between (3, 4) and (6, 8) is 5 units.
Share Phase, Questions 3 through 5

- What is an alternate location for the third point?
- Is the value of $c^2$ a perfect square?
- If the value of $c^2$ is not a perfect square, the value of $c$ lies between which two perfect squares?
- How do negative values for the coordinates affect the distance between the two points?
- Is distance ever described using negative numbers? Explain.

3. $(-6, 4)$ and $(2, -8)$

\[ a^2 + b^2 = c^2 \]
\[ 64 + 144 = c^2 \]
\[ c^2 = 208 \]
\[ c = \sqrt{208} \approx 14.4 \]
The distance between $(-6, 4)$ and $(2, -8)$ is approximately 14.4 units.

4. $(-5, 2)$ and $(-6, 10)$

\[ a^2 + b^2 = c^2 \]
\[ 1 + 64 = c^2 \]
\[ c^2 = 65 \]
\[ c = \sqrt{65} \approx 8.1 \]
The distance between $(-5, 2)$ and $(-6, 10)$ is approximately 8.1 units.
5. \((-1, -4)\) and \((-3, -6)\)

\[a^2 + b^2 = c^2\]
\[2^2 + 2^2 = c^2\]
\[4 + 4 = c^2\]
\[c^2 = 8\]
\[c = \sqrt{8} \approx 2.8\]

The distance between \((-1, -4)\) and \((-3, -6)\) is approximately 2.8 units.

Be prepared to share your solutions and methods.
**Follow Up**

**Assignment**
Use the Assignment for Lesson 2.4 in the Student Assignments book. See the Teacher’s Resources and Assessments book for answers.

**Skills Practice**
Refer to the Skills Practice worksheet for Lesson 2.4 in the Student Assignments book for additional resources. See the Teacher’s Resources and Assessments book for answers.

**Assessment**
See the Assessments provided in the Teacher’s Resources and Assessments book for Chapter 2.

**Check for Students’ Understanding**
Use the coordinate plane provided to answer the questions. One unit represents one kilometer.

Vita’s house is located at point $A (9, 7)$. Her dog wandered away from home, but fortunately, the dog was wearing an identification tag which included Vita’s phone number. Vita received a phone call that the dog was last seen at a location described by point $B (-7, -9)$.

How far did the dog wander from its home?

\[
16^2 + 16^2 = c^2 \\
256 + 256 = c^2 \\
512 = c^2 \\
c = \sqrt{512} \approx 22.6
\]

The dog wandered about 22.6 kilometers from its home.
2.5 Diagonally
Diagonals in Two Dimensions

Learning Goals
In this lesson, you will:
- Use the Pythagorean Theorem to determine the length of diagonals in two-dimensional figures.

Essential Ideas
- The Pythagorean Theorem is used to determine the length of diagonals in two-dimensional figures.
- The Pythagorean Theorem states; if \( a \) and \( b \) are the lengths of the legs of a right triangle and \( c \) is the length of the hypotenuse, then \( a^2 + b^2 = c^2 \).

Texas Essential Knowledge and Skills for Mathematics
Grade 8
(7) Expressions, equations, and relationships. The student applies mathematical process standards to use geometry to solve problems. The student is expected to:

(C) use the Pythagorean Theorem and its converse to solve problems

(D) determine the distance between two points on a coordinate plane using the Pythagorean Theorem
Overview
Students use the Pythagorean Theorem to determine that the diagonals in a rectangle are congruent, and the diagonals in a square are congruent. The Pythagorean Theorem is also used to determine that the diagonals of a trapezoid are only congruent when the trapezoid is isosceles. In the last activity, students will calculate the area of composite figures. The first figure is composed of a rectangle inscribed in a circle, and the second figure is composed of a right triangle and a semi-circle. The formulas for the area of a triangle and area of a circle are used in this activity.
Warm Up

Use the coordinate plane shown to answer each question.

1. What are the coordinates of the vertices of triangle $ABC$?
   - $A(0, 4)$
   - $B(-2, -8)$
   - $C(8, -8)$

2. What is a strategy for determining the length of side $AC$?
   A strategy for determining the length of side $AC$ is to use segment $AC$ as the hypotenuse and draw a right triangle by connecting point $A$ to segment $BC$. Then the Pythagorean Theorem could be used to determine the length of side $AC$.

3. Determine the length of side $AC$.
   
   \[
   12^2 + 8^2 = c^2
   
   144 + 64 = c^2
   
   208 = c^2
   
   c = \sqrt{208} \approx 14.4
   
   The length of side $AC$ is about 14.4 units.
   
4. Can the same strategy be used to determine the length of side $AB$?
   Yes. The same strategy can be used to determine the length of side $AB$.

5. Determine the length of side $AB$.
   
   \[
   12^2 + 2^2 = c^2
   
   144 + 4 = c^2
   
   148 = c^2
   
   c = \sqrt{148} \approx 12.2
   
   The length of side $AB$ is about 12.2 units.
Learning Goal
In this lesson, you will:
- Use the Pythagorean Theorem to determine the length of diagonals in two-dimensional figures.

You have certainly seen signs like this one.

This sign means “no parking.” In fact, a circle with a diagonal line through it (from top left to bottom right) is considered the universal symbol for “no.” This symbol is used on street signs, on packaging, and on clothing labels, to name just a few.

What other examples can you name?
Problem 1

Students draw two diagonals in a rectangle, use the Pythagorean Theorem to determine the length of each diagonal, and conclude that the diagonals of a rectangle are equal in length. Next, students will draw two diagonals in a square, use the Pythagorean Theorem to determine the length of each diagonal, and conclude that the diagonals of a square are equal in length.

Grouping
Have students complete Question 1 with a partner. Then share the responses as a class.

Share Phase, Question 1

- What do you know about the measure of the angles of a rectangle?
- Are the triangles formed by the diagonals of a rectangle right triangles? Why or why not?
- What do you know about the length of the opposite sides of a rectangle?
- What are the lengths of the legs?
- What is the length of the hypotenuse?
- Do you need to use the Pythagorean Theorem to determine the length of the second diagonal in the rectangle? Why or why not?

Problem 1  Diagonals of a Rectangle and a Square

Previously, you have drawn or created many right triangles and used the Pythagorean Theorem to determine side lengths. In this lesson, you will explore the diagonals of various shapes.

1. Rectangle ABCD is shown.

![Diagram of rectangle ABCD with diagonals AC and BD]

a. Draw diagonal AC in rectangle ABCD. Then, determine the length of diagonal AC.

\[ a^2 + b^2 = c^2 \]

\[ 8^2 + 15^2 = c^2 \]

\[ 64 + 225 = c^2 \]

\[ c^2 = 289 \]

\[ c = \sqrt{289} = 17 \]

The length of diagonal AC is 17 feet.

b. Draw diagonal BD in rectangle ABCD. Then, determine the length of diagonal BD.

\[ a^2 + b^2 = c^2 \]

\[ 8^2 + 15^2 = c^2 \]

\[ 64 + 225 = c^2 \]

\[ c^2 = 289 \]

\[ c = \sqrt{289} = 17 \]

The length of diagonal BD is 17 feet.

c. What can you conclude about the diagonals of this rectangle?

The diagonals of this rectangle are equal in length.
Grouping

Have students complete Question 2 with a partner. Then share the responses as a class.

Share Phase, Question 2

- What do you know about the measure of the angles of a square?
- Are the triangles formed by the diagonals of a square right triangles? Why or why not?
- What do you know about the length of the sides of a square?
- What are the lengths of the legs?
- What is the length of the hypotenuse?
- Do you need to use the Pythagorean Theorem to determine the length of the second diagonal in the square? Why or why not?

2. Square ABCD is shown.

a. Draw diagonal AC in square ABCD. Then, determine the length of diagonal AC.

\[ a^2 + b^2 = c^2 \]
\[ 10^2 + 10^2 = c^2 \]
\[ 100 + 100 = c^2 \]
\[ c^2 = 200 \]
\[ c = \sqrt{200} \approx 14.1 \]

The length of diagonal AC is approximately 14.1 meters.

b. Draw diagonal BD in square ABCD. Then, determine the length of diagonal BD.

\[ a^2 + b^2 = c^2 \]
\[ 10^2 + 10^2 = c^2 \]
\[ 100 + 100 = c^2 \]
\[ c^2 = 200 \]
\[ c = \sqrt{200} \approx 14.1 \]

The length of diagonal BD is approximately 14.1 meters.

c. What can you conclude about the diagonals of this square?

The diagonals of this square are equal in length.
Problem 2
Students draw two diagonals in a trapezoid, use the Pythagorean Theorem to determine the length of each diagonal, and conclude that the diagonals of a trapezoid are not equal in length. Next, students will draw two diagonals in an isosceles trapezoid, use the Pythagorean Theorem to determine the length of each diagonal, and conclude that the diagonals of an isosceles trapezoid are equal in length.

Grouping
Have students complete Question 1 with a partner. Then share the responses as a class.

Share Phase, Question 1
• What do you know about the measure of the angles in this trapezoid?
• Are the triangles formed by the diagonals of a trapezoid right triangles? Why or why not?
• Is triangle ABC a right triangle? Why or why not?
• What are the lengths of the legs?
• What is the length of the hypotenuse?
• Can you use the same right triangle to determine the length of the second diagonal? Why or why not?

Problem 2  Diagonals of Trapezoids
1. Graph and label the coordinates of the vertices of trapezoid ABCD. A(1, 2), B(7, 2), C(7, 5), D(3, 5)

a. Draw diagonal AC in trapezoid ABCD.
b. What right triangle can be used to determine the length of diagonal AC?
   Right triangle ABC can be used to determine the length of diagonal AC.

c. Determine the length of diagonal AC.
   \[ a^2 + b^2 = c^2 \]
   \[ 3^2 + 6^2 = c^2 \]
   \[ 9 + 36 = c^2 \]
   \[ c^2 = 45 \]
   \[ c = \sqrt{45} \approx 6.7 \]
   The length of diagonal AC is approximately 6.7 units.

d. Draw diagonal BD in trapezoid ABCD.
e. What right triangle can be used to determine the length of diagonal BD?
   Right triangle BCD can be used to determine the length of diagonal BD.

• Do you need to use the Pythagorean Theorem to determine the length of the second diagonal in the rectangle? Why or why not?
Grouping
Have students complete Question 2 with a partner. Then share the responses as a class.

Share Phase, Question 2, parts (a) through (c)
• What do you know about the measure of the angles in this trapezoid?
• Are the triangles formed by the diagonals of a trapezoid right triangles? Why or why not?
• Can you locate a point to form a right triangle? What are the coordinates of the point?
• What are the lengths of the legs?
• What is the length of the hypotenuse?

f. Determine the length of diagonal $BD$.
\[ a^2 + b^2 = c^2 \]
\[ 3^2 + 4^2 = c^2 \]
\[ 9 + 16 = c^2 \]
\[ c^2 = 25 \]
\[ c = 5 \]
The length of the diagonal $BD$ is 5 units.

g. What can you conclude about the diagonals of this trapezoid?
The diagonals of this trapezoid are not equal in length.

2. Graph and label the coordinates of the vertices of isosceles trapezoid $ABCD$.
$A(1, 2), B(9, 2), C(7, 5), D(3, 5)$

a. Draw diagonal $AC$ in trapezoid $ABCD$.
b. What right triangle can be used to determine the length of diagonal $AC$?
There is no right triangle that can be used to determine the length of diagonal $AC$ in the trapezoid, so I must create a right triangle using points $A$, $C$, and $E(7, 2)$. 

How is this trapezoid different than the first trapezoid you drew?
Share Phase, Question 2, parts (d) through (g)

- Can you use the same right triangle to determine the length of the second diagonal? Why or why not?
- Do you need to use the Pythagorean Theorem to determine the length of the second diagonal in the rectangle? Why or why not?
- Can you locate a second point to form a right triangle? What are the coordinates of the point?
- What is the relationship between the two right triangles you used to determine the length of the diagonals?

c. Determine the length of diagonal AC.
\[ a^2 + b^2 = c^2 \]
\[ 3^2 + 6^2 = c^2 \]
\[ 9 + 36 = c^2 \]
\[ c^2 = 45 \]
\[ c = \sqrt{45} = 6.7 \]
The length of diagonal AC is approximately 6.7 units.

d. Draw diagonal BD in trapezoid ABCD.

e. What right triangle can be used to determine the length of diagonal BD?
There is no right triangle that can be used to determine the length of diagonal BD in the trapezoid, so I must create a right triangle using points B, D, and F(3, 2).

f. Determine the length of diagonal BD.
\[ a^2 + b^2 = c^2 \]
\[ 3^2 + 6^2 = c^2 \]
\[ 9 + 36 = c^2 \]
\[ c^2 = 45 \]
\[ c = \sqrt{45} = 6.7 \]
The length of diagonal BD is approximately 6.7 units.

g. What can you conclude about the diagonals of this isosceles trapezoid?
The diagonals of this isosceles trapezoid are equal in length.
Problem 3
In the first situation, a rectangle is inscribed in a circle and students determine the area of the shaded region by first determining the area of the rectangle and then determining the area of the circle. Subtracting the area of the rectangle from the area of the circle, results in the area of the shaded region. In the second situation, the figure is composed of a right triangle and a semi-circle. Students will determine the area of the entire region by adding the area of the triangle to the area of the semi-circle.

Note
Student may have difficulty seeing the diagonal of the inscribed rectangle is also the diameter of the circle in the first problem, and the hypotenuse of the right triangle is also the diameter of the semi-circle in the second problem. The use of the suggested guiding questions will help, if needed.

Grouping
Have students complete Questions 1 and 2 with a partner. Then share the responses as a class.

Problem 3 Composite Figures
Use your knowledge of right triangles, the Pythagorean Theorem, and areas of shapes to determine the area of each shaded region. Use 3.14 for $\pi$.

1. A rectangle is inscribed in a circle as shown.

Think about how the diagonal of the rectangle relates to the diameter of the circle.

$A = bh$
$A = (6)(10)$
$A = 60$
The area of the rectangle is 60 cm$^2$.

$a^2 + b^2 = c^2$
$6^2 + 10^2 = c^2$
$36 + 100 = c^2$
$c^2 = 136$
$c = $\sqrt{136} = 11.7$
The diameter of the circle is 11.7 cm, so the radius of the circle is 5.85 cm.

$A = \pi r^2$
$A = (3.14)(5.85)^2$
$A = 107.5$
The area of the circle is approximately 107.5 cm$^2$.
$107.5 - 60 = 47.5$
The area of the shaded region is approximately 47.5 cm$^2$. 
Share Phase, 
Question 1

- How do you determine the area of the rectangle?
- Can the Pythagorean Theorem be used to determine the length of the diagonal of the rectangle? Why or why not?
- What is the relationship between the diagonal of the rectangle and the diameter of the circle?
- How do you determine the area of the circle?
- How do you determine the area of the shaded region?

Share Phase, 
Question 2

- What formula is used to determine the area of the right triangle?
- What relationship is the hypotenuse of the right triangle to the diameter of the semi-circle?
- What formula is used to determine the area of a circle?
- How do you determine the area of a semi-circle?
- What operation is used to determine the area of the shaded region?

2. The figure is composed of a right triangle and a semi-circle.

\[ A = \frac{1}{2}bh \]
\[ A = \frac{1}{2}(5)(8) \]
\[ A = 20 \]

The area of the right triangle is 20 mm².

\[ 5^2 + 8^2 = c^2 \]
\[ 25 + 64 = c^2 \]
\[ c^2 = 89 \]
\[ c = \sqrt{89} \approx 9.4 \]

The diameter of the semicircle is 9.4 cm, so the radius of the semicircle is 4.7 cm.

\[ A = \pi r^2 \]
\[ A = (3.14)(4.7)^2 \]
\[ A = 69.4 \]
\[ \frac{1}{2} A = 34.7 \]

The area of the semi-circle is approximately 34.7 mm².

\[ 34.7 + 20 = 54.7 \]

The area of the shaded region is approximately 54.7 square millimeters.

Be prepared to share your solutions and methods.
Follow Up

Assignment
Use the Assignment for Lesson 2.5 in the Student Assignments book. See the Teacher’s Resources and Assessments book for answers.

Skills Practice
Refer to the Skills Practice worksheet for Lesson 2.5 in the Student Assignments book for additional resources. See the Teacher's Resources and Assessments book for answers.

Assessment
See the Assessments provided in the Teacher’s Resources and Assessments book for Chapter 2.

Check for Students’ Understanding
Use the parallelogram graphed on the coordinate plane to answer each question.

1. What are the coordinates of the vertices of parallelogram $ABCD$?
   $A (-4, 2)$, $B (8, 2)$, $C (4, -4)$, $D (-8, -4)$

2. What is a strategy for determining the length of diagonal $AC$?
   A strategy for determining the length of diagonal $AC$ is to use segment $AC$ as the hypotenuse and draw a right triangle by connecting point $A$ to segment $CD$. Then the Pythagorean Theorem could be used to determine the length of diagonal $AC$.

3. Determine the length of diagonal $AC$.
   $6^2 + 8^2 = c^2$
   $36 + 64 = c^2$
   $100 = c^2$
   $c = \sqrt{100} = 10$
   The length of diagonal $AC$ is 10 units.
4. Can the same strategy be used to determine the length of diagonal $BD$?
   Yes. The same strategy can be used to determine the length of diagonal $BD$.

5. Determine the length of diagonal $BD$.
   \[ 6^2 + 8^2 = c^2 \]
   \[ 36 + 64 = c^2 \]
   \[ 100 = c^2 \]
   \[ c = \sqrt{100} = 10 \]
   The length of diagonal $BD$ is 10 units.

6. What can you conclude about the length of the diagonals of a parallelogram?
   The length of the diagonals of a parallelogram are equal.
Two Dimensions Meet Three Dimensions
Diagonals in Three Dimensions

Learning Goals
In this lesson, you will:
- Use the Pythagorean Theorem to determine the length of a diagonal of a solid.
- Use a formula to determine the length of a diagonal of a rectangular solid given the lengths of three perpendicular edges.
- Use a formula to determine the length of a diagonal of a rectangular solid given the diagonal measurements of three perpendicular sides.

Essential Ideas
- The Pythagorean Theorem is used to determine the length of diagonals in three-dimensional figures.
- A formula used to determine the length of a three-dimensional diagonal of a rectangular solid is $d^2 = l^2 + w^2 + h^2$, where $d$ is the length of the three-dimensional diagonal, and $l$, $w$, and $h$ are the three-dimensions, length, width, and height of the solid.
- A formula used to determine the length of a three-dimensional diagonal of a rectangular solid is $d^2 = \frac{1}{2}(x^2 + y^2 + z^2)$, where $d$ is the length of the three-dimensional diagonal, and $x$, $y$, and $z$ are the lengths of the diagonals of each unique face of the solid.

Texas Essential Knowledge and Skills for Mathematics
Grade 8
(7) Expressions, equations, and relationships. The student applies mathematical process standards to use geometry to solve problems. The student is expected to:
(C) use the Pythagorean Theorem and its converse to solve problems.
Overview

Students use the Pythagorean Theorem to determine the length of a three-dimensional diagonal of a rectangular solid. Several rectangular solids are given and students sketch a three-dimensional diagonal in each solid. Students will use a formula to compute the length of a three-dimensional diagonal. The formula involves taking the square root of the sum of the squares of the lengths of three perpendicular edges of the rectangular solid. A second formula to determine the length of a three-dimensional diagonal of a rectangular solid is introduced. This formula involves taking the square root of one-half the sum of the squares of the diagonals of each unique face of the rectangular solid.
Warm Up

1. Draw dashed lines to show all faces of the rectangular solid.

2. Imagine that the rectangular solid is a room. An ant is on the floor situated at point A. Describe the shortest path the ant can crawl to get to point B in the corner of the ceiling.
   
   The shortest path the ant could crawl to get from point A to point B would be across the floor diagonally to the furthest corner of the floor and then straight up the wall to point B.

3. Suppose it isn’t really an ant at all, it’s a fly! Describe the shortest path the fly can fly to get from point A to point B.
   
   The shortest path the fly could fly to get from point A to point B would a straight line connecting points A and B.

4. If the ant’s path and the fly’s path were connected, what figure would it form?
   
   If the ant’s path and the fly’s path were connected, it would form a right triangle.
Harry Houdini was one of the most famous escapologists in history. What is an escapologist? He or she is a person who is an expert at escaping from restraints—like handcuffs, cages, barrels, fish tanks, and boxes.

On July 7, 1912, Houdini performed an amazing box escape. He was handcuffed, and his legs were shackled together. He was then placed in a box which was nailed shut, roped, weighed down with 200 pounds of lead, and then lowered into the East River in New York!

Houdini managed to escape in less than a minute. But he was a professional. So don't try to become an escapologist at home!
Problem 1
Students compare the diagonal of a rectangle to the diagonal of a rectangular solid, distinguishing between two and three dimensions. To determine the three-dimensional diagonal, they first use the Pythagorean Theorem to calculate the length of the diagonal across the bottom face of the solid, and then use that length in addition to the height of the solid in the Pythagorean Theorem again, to compute the length of the three-dimensional diagonal.

Grouping
- Ask a student to read the information before Question 1 aloud. Discuss the context as a class.
- Have students complete Questions 1 through 11 with a partner. Then share the responses as a class.

Share Phase, Questions 1 through 11
- How many planes are used to draw a rectangle?
- How are the diagonals drawn in a rectangle?
- How many different diagonals are in a rectangle?
- How many planes are used to draw a rectangular solid?
- How are the diagonals drawn in a rectangular solid?
- How many faces are on a rectangular solid?

Problem 1  A Box of Roses
A rectangular box of long-stem roses is 18 inches in length, 6 inches in width, and 4 inches in height.

Without bending a long-stem rose, you are to determine the maximum length of a rose that will fit into the box.

1. What makes this problem different from all of the previous applications of the Pythagorean Theorem? I have always applied the Pythagorean Theorem within a two-dimensional plane. This problem will require me to use the Pythagorean Theorem in a three-dimensional situation.

2. Compare a two-dimensional diagonal to a three-dimensional diagonal. Describe the similarities and differences.

A two-dimensional diagonal and a three-dimensional diagonal are both line segments that connect any vertex to another vertex that does not share a face or an edge. A two-dimensional diagonal shares the same plane as the sides on which the figure is drawn. A three-dimensional diagonal does not share a plane with any side on which the figure is drawn.

- How many edges are on a rectangular solid?
- How many different diagonals can be drawn in a rectangular solid?
- How would you describe the location of the right triangle used to determine the length of a three-dimensional diagonal?
3. Which diagonal represents the maximum length of a rose that can fit into a box?
   The 3-D diagonal represents the maximum length of a rose that could fit into a box.

4. Draw all of the sides in the rectangular solid you cannot see using dotted lines.

5. Draw a three-dimensional diagonal in the rectangular solid shown.

6. Let’s consider that the three-dimensional diagonal you drew in the rectangular solid is also the hypotenuse of a right triangle. If a vertical edge is one of the legs of that right triangle, where is the second leg of that same right triangle?
   The second leg is a diagonal drawn on the bottom of the box.

7. Draw the second leg using a dotted line. Then lightly shade the right triangle.

8. Determine the length of the second leg you drew.
   \[ d^2 = 18^2 + 6^2 \]
   \[ d^2 = 324 + 36 \]
   \[ d^2 = 360 \]
   \[ d = \sqrt{360} \approx 18.97 \]
   The length of the second leg is approximately 18.97 inches.

9. Determine the length of the three-dimensional diagonal.
   \[ d^2 = 18.97^2 + 4^2 \]
   \[ d^2 = 359.66 + 16 \]
   \[ d^2 = 375.96 \]
   \[ d = \sqrt{375.96} \approx 19.39 \]
   The three-dimensional diagonal is approximately 19.39 inches long.

10. What does the length of the three-dimensional diagonal represent in terms of this problem situation.
    The maximum length of a rose that will fit into a 18" × 6" × 4" box is approximately 19.39 inches.
Problem 2
Students practice drawing the sides and the three-dimensional diagonal in rectangular solids.

Grouping
Have students complete Questions 1 through 6 with a partner. Then share the responses as a class.

Share Phase, Questions 1 through 6
- Is there more than one way to draw the diagonal in this rectangular solid? Explain.
- Do you think all of the three-dimensional diagonals in this rectangular solid are the same length? Why or why not?
- Which faces of the rectangular solid are involved in computing the length of the three-dimensional diagonal?

11. Describe how the Pythagorean Theorem was used to solve this problem.
   I formed a right triangle on the plane in which the three-dimensional diagonal was drawn such that it was the hypotenuse. However, I did not know the length of a leg of this triangle. I first applied the Pythagorean Theorem to determine the length of a leg using the plane on the bottom of the rectangular solid, and then I used the Pythagorean Theorem a second time to determine the length of the three-dimensional diagonal.

Problem 2 Drawing Diagonals
Draw all of the sides you cannot see in each rectangular solid using dotted lines. Then draw a three-dimensional diagonal using a solid line.

1. 
2. 
3. 
4. 
5. 
6. 

How many three-dimensional diagonals can be drawn in each figure?
Problem 3

Six rectangular solids and their dimensions are given. Students use the Pythagorean Theorem to calculate the length of a three-dimensional diagonal in each solid.

Grouping

Have students complete Questions 1 through 6 with a partner. Then share the responses as a class.

Share Phase, Questions 1 through 6

- What dimensions of the rectangular solid are used to calculate the length of the diagonal of the bottom face?
- What dimension of the rectangular solid is used to calculate the length of the three-dimensional diagonal?
- How many times was the Pythagorean Theorem used in each problem?
- Why must the Pythagorean Theorem be used twice?

Problem 3 Applying the Pythagorean Theorem

Determine the length of the diagonal of each rectangular solid.

1. Length of second leg: 6 in. 
   Length of diagonal:
   \[ d^2 = 6^2 + 4^2 \]
   \[ = 36 + 16 \]
   \[ d = \sqrt{52} \approx 7.21 \]
   The length of the diagonal of the rectangular solid is about 7.21 inches.

2. Length of second leg: 4 m
   Length of diagonal:
   \[ d^2 = 8^2 + 7^2 \]
   \[ = 64 + 16 \]
   \[ d = \sqrt{80} \approx 8.94 \]
   The length of the diagonal of the rectangular solid is about 8.94 meters.
3. 

Length of second leg: 
\[ d^2 = 10^2 + 6^2 \]
\[ = 100 + 36 \]
\[ d = \sqrt{136} \approx 11.66 \]

Length of diagonal: 
\[ d^2 = 15^2 \]
\[ = 225 \]
\[ d = \sqrt{225} = 15 \]

The length of the diagonal of the rectangular solid is about 15.0 centimeters.

4. 

Length of second leg: 
\[ d^2 = 5^2 + 7^2 \]
\[ = 25 + 49 \]
\[ d = \sqrt{74} \approx 8.60 \]

Length of diagonal: 
\[ d^2 = 6.60^2 + 7^2 \]
\[ = 43.56 + 49 \]
\[ d = \sqrt{92.56} \approx 11.09 \]

The length of the diagonal of the rectangular solid is about 11.09 yards.
5. \[\text{Length of second leg:} \quad d^2 = 3^2 + 15^2 \]
\[\Rightarrow 9 + 225 = 234 \]
\[d = \sqrt{234} \approx 15.30 \]

The length of the diagonal of the rectangular solid is about 16.10 inches.

6. \[\text{Length of second leg:} \quad d^2 = 2^2 + 12^2 \]
\[\Rightarrow 4 + 144 = 148 \]
\[d = \sqrt{148} \approx 12.17 \]

The length of the diagonal of the rectangular solid is about 12.33 feet.
Problem 4

A formula is introduced to determine the length of a three-dimensional diagonal of a rectangular solid. The formula is \( d^2 = l^2 + w^2 + h^2 \), where \( d \) is the length of the three-dimensional diagonal, and \( l \), \( w \), and \( h \) are the three dimensions, length, width, and height of the solid. Students use this formula to recalculate the answers to Problem 3. They conclude the answers are the same as previously calculated. A second formula is introduced to determine the length of a three-dimensional diagonal of a rectangular solid. The formula is \( d^2 = \frac{1}{2}(x^2 + y^2 + z^2) \), where \( d \) is the length of the three-dimensional diagonal, and \( x \), \( y \), and \( z \) are the lengths of the diagonals of each unique face of the solid.

Grouping

Have students complete Questions 1 and 2 with a partner. Then share the responses as a class.

Share Phase, Questions 1 and 2

- What is the length of the rectangular solid?
- What is the width of the rectangular solid?
- What is the height of the rectangular solid?
- How are the length, width, and height used to calculate the length of a three-dimensional diagonal?

Problem 4  Student Discovery

1. Norton thought he knew a shortcut for determining the length of a three-dimensional diagonal. He said, “All you have to do is calculate the sum of the squares of the rectangular solids’ 3 perpendicular edges (the length, the width, and the height) and that sum would be equivalent to the square of the three-dimensional diagonal.” Does this work? Use the rectangular solid in Problem 1 to determine if Norton is correct. Explain your reasoning.

\[
SD^2 = 4^2 + 6^2 + 18^2 \\
SD^2 = 16 + 36 + 324 \\
SD^2 = 376 \\
SD = \sqrt{376} = 19.39 \\
Norton is correct.
\]

2. Use Norton’s strategy to calculate the length of the diagonals of each rectangular solid in Problem 3. How do these answers compare to the answers in Problem 3? The answers are the same.
**Grouping**

- Ask a student to read the information prior to Question 3 aloud. Discuss the example as a class.
- Have students complete Question 3 with a partner. Then share the responses as a class.

**Share Phase, Question 3**

- Do you think it is easier to use this formula or the Pythagorean Theorem twice to determine the length of a three-dimensional diagonal? Why?
- Does this formula use the Pythagorean Theorem?
- Why do you think this formula works?
- Can this formula be used to determine the length of a three-dimensional diagonal in any rectangular solid? Explain.

The square of a three-dimensional diagonal is equal to the sum of the squares of each dimension of the rectangular solid.

\[ d^2 = l^2 + w^2 + h^2 \]

\[ d = \sqrt{l^2 + w^2 + h^2} \]

3. Use the formula \( d = \sqrt{l^2 + w^2 + h^2} \) to determine the length of a three-dimensional diagonal of the rectangular prism shown.

\[ d = \sqrt{11^2 + 6^2 + 4^2} \]
\[ = \sqrt{121 + 36 + 16} \]
\[ = \sqrt{173} \approx 13.15 \]

The length of a three-dimensional diagonal of the rectangular prism is about 13.15 centimeters.
Grouping

- Ask a student to read the information prior to Question 4 aloud. Discuss the worked example as a class.
- Have students complete Questions 4 through 6 with a partner. Then share the responses as a class.

Discuss Phase, Worked Example

- How is this formula different than the previous formula?
- How is this formula similar to the previous formula?
- How do you know when to use which formula?
- Which formula is easiest to work with? Why?

If you know the diagonal lengths of each face of a rectangular solid, you can determine the length of a three-dimensional diagonal.

Let \( d \) represent the length of a three-dimensional diagonal.

\[
d^2 = \frac{1}{2}(\text{sum of the squares of the diagonals of each unique face})
\]

\[
d^2 = \frac{1}{2}(18^2 + 15^2 + 12^2)
\]

\[
d^2 = \frac{1}{2}(324 + 225 + 144)
\]

\[
d^2 = 346.5
\]

\[
d = \sqrt{346.5}
\]

\[
d = 18.6
\]

The length of the three-dimensional diagonal of this rectangular prism is about 18.6 inches.

Use your knowledge of diagonals and the two formulas given to answer each question.

4. A rectangular box has a length of 6 feet and a width of 2 feet. The length of a three-dimensional diagonal of the box is 7 feet. What is the height of the box?

\[
d^2 = l^2 + w^2 + h^2
\]

\[
7^2 = 6^2 + 2^2 + h^2
\]

\[
49 = 36 + 4 + h^2
\]

\[
9 = h^2
\]

\[
h = 3
\]

The height of the box is 3 feet.
5. The length of the diagonal across the front of a rectangular box is 20 inches, and the length of the diagonal across the side of the box is 15 inches. The length of a three-dimensional diagonal of the box is 23 inches. What is the length of the diagonal across the top of the box?

\[ d^2 = \frac{1}{2} (d_1^2 + d_2^2 + d_3^2) \]

\[ 23^2 = \frac{1}{2} (20^2 + 15^2 + d_3^2) \]

\[ 529 = \frac{1}{2} (400 + 225 + d_3^2) \]

\[ 1058 = 625 + d_3^2 \]

\[ 433 = d_3^2 \]

\[ 20.81 = d_3 \]

The length of the diagonal across the top of the box is about 20.81 inches.

6. Pablo is packing for a business trip. He is almost finished packing when he realizes that he forgot to pack his umbrella. Before Pablo takes the time to repack his suitcase, he wants to know if the umbrella will fit in the suitcase. His suitcase is in the shape of a rectangular prism and has a length of 2 feet, a width of 1.5 feet, and a height of 0.75 foot. The umbrella is 30 inches long. Will the umbrella fit in Pablo’s suitcase? Explain your reasoning.

Length of three-dimensional diagonal of suitcase:

\[ d = \sqrt{2^2 + 1.5^2 + 0.75^2} \]

\[ = \sqrt{4 + 2.25 + 0.5625} \]

\[ = \sqrt{6.8125} \]

\[ = 2.61 \text{ ft} \]

Length of umbrella:

(30 in.) \( \frac{1 \text{ ft}}{12 \text{ in.}} \) = 2.5 ft

The umbrella is 2.5 feet long, and the length of the three-dimensional diagonal of the suitcase is approximately 2.61 feet. So, the umbrella will fit in Pablo’s suitcase.

Be prepared to share your solutions and methods.
Check for Students' Understanding

A rectangular room is 10' \times 16' \times 8' feet.

An ant crawls from point $A$ to point $B$ taking the shortest path.

A fly flies from point $A$ to point $B$ taking the shortest path.

1. Whose path was the shortest?
   
   The fly's path is shorter than the ant's path.

2. How much shorter is the shortest path?

   $10^2 + 16^2 = c^2$
   $100 + 256 = c^2$
   $356 = c^2$
   $c = \sqrt{356} \approx 18.9$

   $18.9 + 8 = 26.9$
   The ant's path is about 26.9 feet.

   $d^2 = 8^2 + 10^2 + 16^2$
   $d^2 = 64 + 100 + 256$
   $d^2 = 420$
   $d = \sqrt{420} \approx 20.5$

   The fly's path is 6.4 feet shorter than the ant's path.
Chapter 2  Summary

Key Terms
- right triangle (2.1)
- right angle (2.1)
- leg (2.1)
- hypotenuse (2.1)
- diagonal of a square (2.1)
- Pythagorean Theorem (2.1)
- theorem (2.1)
- postulate (2.1)
- proof (2.1)
- converse (2.2)
- Converse of the Pythagorean Theorem (2.2)
- Pythagorean triple (2.2)

2.1 Applying the Pythagorean Theorem

A right triangle is a triangle with a right angle. A right angle is an angle with a measure of 90° and is indicated by a square drawn at the corner formed by the angle. A leg of a right triangle is either of the two shorter sides. Together, the two legs form the right angle of a right triangle. The hypotenuse of a right triangle is the longest side and is opposite the right angle. The Pythagorean Theorem states that if $a$ and $b$ are the lengths of the legs of a right triangle and $c$ is the length of the hypotenuse, then $a^2 + b^2 = c^2$.

Example

Determine the unknown side length of the triangle.

\[
\begin{align*}
4^2 + 8^2 &= c^2 \\
16 + 64 &= c^2 \\
80 &= c^2 \\
\sqrt{80} &= c \\
c &= 8.9
\end{align*}
\]

The unknown side length of the triangle is about 8.9 units.

Whoa! My brain got a workout with that chapter. I better get a good night’s sleep to recover from all that hard work!
Applying the Converse of the Pythagorean Theorem

The Converse of the Pythagorean Theorem states that if \( a, b, \) and \( c \) are the side lengths of a triangle and \( a^2 + b^2 = c^2 \), then the triangle is a right triangle.

**Example**

Determine whether a triangle with side lengths 5, 9, and 10 is a right triangle.

\[
a^2 + b^2 = c^2
\]
\[
5^2 + 9^2 \neq 10^2
\]
\[
25 + 81 \neq 100
\]
\[
106 \neq 100
\]

A triangle with side lengths 5, 9, and 10 is not a right triangle because \( 5^2 + 9^2 \neq 10^2 \).

Applying the Pythagorean Theorem to Solve Real-World Problems

The Pythagorean Theorem can be used to solve a variety of real-world problems which can be represented by right triangles.

**Example**

An escalator in a department store carries customers from the first floor to the second floor. Determine the distance between the two floors.

\[
a^2 + b^2 = c^2
\]
\[
30^2 + b^2 = 36^2
\]
\[
900 + b^2 = 1296
\]
\[
b^2 = 396
\]
\[
b = \sqrt{396}
\]
\[
b \approx 19.90
\]

The distance between the two floors is 19.90 feet.
Determine the Distance Between Two Points in a Coordinate System

The distance between two points, which do not lie on the same horizontal or vertical line, on a coordinate plane can be determined using the Pythagorean Theorem.

Example

Determine the distance between points \((-5, 3)\) and \((7, -2)\).

A line segment is drawn between the two points to represent the hypotenuse of a right triangle. Two line segments are drawn (one horizontal and one vertical) to represent the legs of the right triangle. The lengths of the legs are 5 units and 12 units.

\[
\begin{align*}
5^2 + 12^2 &= c^2 \\
25 + 144 &= c^2 \\
c^2 &= 169 \\
c &= \sqrt{169} \\
c &= 13
\end{align*}
\]

The distance between \((-5, 3)\) and \((7, -2)\) is 13 units.
Determining the Lengths of Diagonals Using the Pythagorean Theorem

The Pythagorean Theorem can be a useful tool for determining the length of a diagonal in a two-dimensional figure.

Example

Determine the area of the shaded region.

The area of the square is:

\[ A = s^2 \]
\[ A = 7^2 \]
\[ A = 49 \text{ square inches} \]

The diagonal of the square is the same length as the diameter of the circle. The diagonal of the square can be determined using the Pythagorean Theorem.

\[ a^2 + b^2 = c^2 \]
\[ 7^2 + 7^2 = c^2 \]
\[ 49 + 49 = c^2 \]
\[ c^2 = 98 \]
\[ c = \sqrt{98} \]
\[ c \approx 9.90 \text{ inches} \]

So, the radius of the circle is \( \frac{1}{2}(9.90) = 4.95 \text{ inches} \)

The area of the circle is:

\[ A = \pi r^2 \]
\[ A = (3.14)(4.95)^2 \]
\[ A \approx 76.94 \text{ square inches} \]

The area of the shaded region is 76.94 \( - 49 \approx 27.94 \text{ square inches} \).
2.6 Determining the Lengths of Diagonals in Three-Dimensional Solids

The Pythagorean Theorem can be used to determine the length of a diagonal in a geometric solid. An alternate formula derived from the Pythagorean Theorem can also be used to determine the length of a diagonal in a geometric solid. In a right rectangular prism with length \( \ell \), width \( w \), height \( h \), and diagonal length \( d \), \( d^2 = \ell^2 + w^2 + h^2 \).

**Example**

Determine the length of a diagonal in a right rectangular prism with a length of 4 feet, a width of 3 feet, and a height of 2 feet.

The diagonal is the hypotenuse of a triangle with one leg being the front left edge of the prism and the other leg being the diagonal of the bottom face.

The length of the diagonal of the bottom face is:

\[
\begin{align*}
    a^2 + b^2 &= c^2 \\
    3^2 + 4^2 &= c^2 \\
    9 + 16 &= c^2 \\
    c^2 &= 25 \\
    c &= \sqrt{25} \\
    c &= 5 \text{ feet}
\end{align*}
\]

The length of the prism’s diagonal is:

\[
\begin{align*}
    a^2 + b^2 &= c^2 \\
    2^2 + 5^2 &= c^2 \\
    4 + 25 &= c^2 \\
    c^2 &= 29 \\
    c &= \sqrt{29} \\
    c &= 5.39 \text{ feet}
\end{align*}
\]

Using the alternate formula, the length of the prism’s diagonal is:

\[
\begin{align*}
    d^2 &= \ell^2 + w^2 + h^2 \\
    d^2 &= 4^2 + 3^2 + 2^2 \\
    d^2 &= 16 + 9 + 4 \\
    d^2 &= 29 \\
    d &= \sqrt{29} \\
    d &\approx 5.39 \text{ feet}
\end{align*}
\]
Soon You Will Determine the Right Triangle Connection
The Pythagorean Theorem

1. Lamar goes shopping for a new flat-panel television. A television is usually described by the length of the screen's diagonal. He finds a great deal on a 42-inch display model.

   a. If the screen's height is 21 inches, what is the width of the screen? Show your work. Round your answer to the nearest tenth.
Lesson 2.1 Assignment

b. The border around the screen is 2 inches. What are the dimensions of the television, including the border? Show your work.

c. How long is the diagonal of the television, including the border? Show your work. Round your answer to the nearest tenth.
2. Lamar sells his old television in his neighborhood's garage sale. It has a rectangular screen with a diagonal measure of 27 inches. A potential buyer is concerned about the television fitting in the 24-inch square opening of his entertainment center.

a. What is the width of the television's screen? Show your work. Round your answer to the nearest tenth.

b. Will the television fit in the buyer's entertainment center? Explain your reasoning.
Lesson 2.2 Assignment

NAME ___________________________ DATE ______________________

Can That Be Right?
The Converse of the Pythagorean Theorem

1. Elena has received grant money to open a local community center. She wants to save as much of the money as possible for programs. She will be doing many of the improvements herself to the old building she has rented. While touring the building to make her project list, she uses a tape measure to check whether floors, doorways, and walls are square, meaning that they meet at right angles.

   a. Elena measures the lobby of the building for new laminate flooring. The length is 30 feet, the width is 16 feet, and the diagonal is 34 feet. Is the room square? Show your work.

   b. Can Elena use the edges of the room as a guide to start laying the boards of laminate flooring? Explain.
c. The landing outside the main entrance of the building does not have a railing. Elena wants to install railing around the landing to make it safer. The length of the landing is 12 feet, the width is 9 feet, and the diagonal is 14 feet. Is the landing square? Show your work.

d. Do you think Elena should install the new railing herself or hire a professional? Explain.
Lesson 2.2 Assignment

NAME ______________________________ DATE __________________

e. Elena needs to order a new door for her office. The width of the door frame is 3 feet, the height is 8 feet, and the diagonal is $8\frac{5}{8}$ feet. Is the door frame square? Show your work.
f. The sign that will be mounted to the outside of the building is a rectangle that is 9 feet by 12 feet. The largest doorway into the building is 4 feet wide and 8 feet high.

What is the diagonal measurement of the doorway? Show your work.

g. Does Elena have to mount the sign the day it is delivered or can she store it inside the building until she is ready? Explain your answer.
Lesson 2.3 Assignment

NAME __________________________________________ DATE _____________

Pythagoras to the Rescue
Solving for Unknown Lengths

1. A bird leaves its nest and flies 2 miles due north, then 3 miles due east, then 4 miles due north, and then 5 miles due east, finally reaching the ocean. The diagram shows the bird’s path.

   a. Use a straightedge to draw a straight line connecting the nest and the ocean in the diagram.

   b. What does this straight line represent?

   c. Extend the vertical line from the nest and the horizontal line from the ocean until they intersect.

   d. Do the lines you drew in part (c) intersect at a right angle? Explain your reasoning.
Lesson 2.3  Assignment

e. Is the large triangle formed a right triangle? Explain your reasoning.

f. Can you use the triangle formed in part (d) to calculate the distance from the nest to the ocean? Explain your reasoning.

g. Calculate the distance from the nest to the ocean. Show your work.
Lesson 2.4  Assignment

Meeting Friends
The Distance Between Two Points in a Coordinate System

Ben is playing soccer with his friends Abby and Clay. The grid shows their locations on the soccer field. Each grid square represents a square that is 2 meters long and 2 meters wide.

1. What are the coordinates of the location of each player?

2. How far does Abby have to kick the ball to reach Clay?

3. How far does Ben have to kick the ball to reach Abby?
4. How far does Ben have to kick the ball to reach Clay?

5. Graph and connect each pair of points on the grid below. Then calculate the distance between each pair of points.

Points (–8, 3) and (–8, 9); Distance = ______

Points (–6, 8) and (–1, 8); Distance = ______

Points (8, –7) and (–4, –7); Distance = ______

Points (8, 8) and (8, –2); Distance = ______
Lesson 2.4 Assignment

NAME ____________________________________________ DATE __________________________

a. Describe the method that you used to determine the distance between each pair of points.

b. Suppose that you were only given the coordinates of the points and did not graph them.
   Describe the method that you would use to calculate the distance between each pair of points.

6. Use the grid to graph and connect the given set of three points. Then, calculate the distances between the points.
   a. (4, 1), (2, 1), and (4, 4)
b. \((1, -4), (1, 1), \text{ and } (-2, -4)\)

c. Describe the method that you used to calculate the distances between the points.
Diagonally
Diagonals in Two Dimensions

1. Use figure $ABCD$ to answer the questions.

   a. Is figure $ABCD$ a kite? Explain your reasoning.

   b. Use your straightedge to draw in diagonals $AC$ and $BD$. Mark the point where the diagonals intersect as point $E$.

   c. Use your protractor to measure the angles formed where the diagonals intersect at point $E$. Record your measure on the figure. What conclusion can you draw about the diagonals of a kite? Explain your reasoning.
d. In a kite, the longer diagonal bisects the shorter diagonal. So, $\overline{AC}$ bisects $\overline{BD}$. Suppose $\overline{BD}$ has a measure of 16 millimeters. What are the measures of $\overline{BE}$ and $\overline{ED}$? Explain your reasoning.

e. Write the measures of $\overline{BE}$ and $\overline{ED}$ on the figure. What kind of triangle is $\triangle BEC$? Explain your reasoning.

f. Determine the length of $\overline{EC}$. Show your work.
g. What kind of triangle is ∆BEA? Explain your reasoning.

h. Determine the length of \( \overline{AE} \). Show your work.

i. Calculate the length of diagonal \( \overline{AC} \). Explain how you determined your answer.

j. Does the shorter diagonal of a kite bisect the longer diagonal in the kite? Explain your reasoning.
Two Dimensions Meet Three Dimensions
Diagonals in Three Dimensions

1. David wants to mail a golf club to his brother. The golf club is 3 feet 6 inches long. David goes to Ships-4-Less shipping company to mail the golf club. His shipping expense will be determined by the weight of the package and its size. David would like to ship the golf club in the smallest possible box to save money. The figures below show his 3 options for boxes in which to ship the golf club.

a. Which box do you think David should choose? Explain your reasoning.

b. How can you determine whether the golf club will fit in each of the boxes? Explain your reasoning.

c. Calculate the diagonal of box A. Show your work.
d. Calculate the diagonal of box B. Show your work.

e. Calculate the diagonal of box C. Show your work.

f. Which of the boxes will the golf club fit in? Explain your reasoning.
g. The size of the box is determined by its volume. Calculate the volume of the boxes that the golf club will fit in. Show your work.

h. Which box do you think David should choose? Explain your reasoning.